HyperSoft Set Methods in Engineering

Journal Homepage: sciencesforce.com/hsse

HyperSoft Set Meth. Eng. Vol. 2 (2024) 38–61

Paper Type: Original Article

Applications of Spherical Fuzzy Hypersoft Sets in Decision Support Systems and Beyond

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Received: 23 Feb 2024 **Revised**: 27 May 2024 **Accepted**: 26 Jun 2024 **Published**: 28 Jun 2024

Abstract

This article develops the idea of a spherical fuzzy hypersoft set through practical application. This study's scope may encompass a broad variety of applications in numerous mathematical science domains. The proposed model incorporates the features of most relevant existing models and overcomes their drawbacks by introducing a novel approximation technique. The Spherical Fuzzy Hypersoft Set (SFHSS) is an extension of traditional fuzzy sets that includes spherical functions and hypersoft sets. By including hypersoft sets into SFHSS, one can handle situations in which membership degrees are not known with precision or in which gradual transitions between membership grades are required. Many domains where complexity and uncertainty are prevalent have applications for SFHSS. Examples include image processing, pattern recognition, data mining, expert systems, machine learning, and decision-making. Due to its enhanced representational powers, real-world problems can be simulated and examined using SFHSS. Overall, for data modeling and analysis, spherical fuzzy hypersoft sets provide a useful framework for handling complexity, imprecision, and uncertainty. They offer a more flexible and expressive means of representing data, which improves decision-making and problem-solving abilities in a wide range of contexts.

Keywords: Hypersoft Sets, Spherical Fuzzy Set, Decision Making, Optimization Theory, Spherical Fuzzy Hypersoft Set, Multi-Criteria Decision Making.

1 | Introduction

The problems we face daily can be resolved in a variety of ways. However, there are unknowns. Soft sets require a flexible approach in set theory to handle these kinds of problems and reduce ambiguity. The core idea behind soft set theory [1] entails parameterizing uncertainty and then describing the uncertainties with the help of that parameterization. Soft set theory is hence an extremely adaptable and versatile method that can deal with a variety of uncertainty modeling scenarios. But remember that the correctness of the parameter information provided and the choice of an appropriate parameter set determines how effectively soft set-based models function [2]. A fuzzy soft set is a substantial extension of soft set theory that goes beyond regular sets by allowing elements to have degrees of membership, including ambiguity and uncertainty to enable flexible membership, and having values between 0 and 1. Healthcare uses for fuzzy logic include patient monitoring and diagnostic systems [3]. Image processing methods, which help with things like edge detection, segmentation, and object recognition, can be enhanced with the help of fuzzy set theory. Fuzzy logic outperforms conventional methods in handling ambiguity and uncertainty in digital images [4, 5].

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https://doi.org/10.61356/j.hsse.2024.2312
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Type-2 Fuzzy Sets [6], Interval-Valued Fuzzy Sets [7], Intuitionistic Fuzzy Sets, Neutrosophic Sets, Hesitant Fuzzy Sets, Pythagorean Fuzzy Sets, Rough Fuzzy Sets [8], and Picture Fuzzy Sets [9] are various extensions of classical crisp sets that capture different aspects of uncertainty, imprecision, and complexity in data representation and decision-making emerge as extensions of fuzzy sets.

1.1 | Background and Motivation

When it comes to making decisions and solving problems, ambiguity is a constant obstacle. Approaches that may consider the imprecision, vagueness, and ambiguity in data and information are frequently required due to the complexities of our world. Since its introduction as a mathematical framework by Molodtsov in 1999, soft set theory has become a viable approach to dealing with these kinds of uncertainty. Soft sets are a flexible tool for modeling different types of uncertainty and indeterminacy because they allow elements to have varying degrees of membership.

But when problem domains get more complex and varied, the demand for a more advanced methodology that can handle a wider range of uncertain scenarios emerges. As a result, the theory of soft sets is significantly expanded to include the idea of spherical fuzzy hypersoft sets. Spherical fuzzy hypersoft sets are based on the idea of parameterizing uncertainty to make it easier to describe uncertainty and make the modeling framework more flexible.

The idea of a soft set was first presented as an expansion of classical set theory in Molodtsov's groundbreaking work, which is where soft set theory got its start. Soft sets let items have different levels of membership in a set, which makes them a natural and obvious approach to describe uncertainty. Soft set theory finds uses in fields including data mining, expert systems, and decision-making after this groundbreaking first step toward capturing uncertainty.

Soft set theory has inherent limits when dealing with more intricate and diverse uncertainty scenarios, even while it works well for some kinds of uncertainty. When the degree of membership is binary (0 or 1), traditional soft sets are excellent at expressing crisp uncertainty; however, they have trouble effectively representing scenarios that contain gradations of uncertainty. Soft-set theory has been extended and improved upon, allowing it to account for a wider range of uncertainty manifestations, which has motivated researchers to explore.

1.2 | Aim and Scope of the Study

The paper's primary objectives are to introduce and explore the idea of spherical fuzzy hypersoft sets as a novel approach to modeling uncertainty. Through the combination of soft set flexibility and fuzzy set adaptability, these sets create a more sophisticated and flexible framework for managing uncertainty. By parameterizing uncertainty and applying fuzzy logic principles, spherical fuzzy hypersoft sets offer a workable solution to practical issues demanding sophisticated uncertainty modeling techniques.

The goal of this work is to thoroughly investigate the intricacies of spherical fuzzy hypersoft sets. Our goal is to provide a thorough understanding of the potential and importance of this addition to soft-set theory through a rigorous exploration of fundamental concepts, important operations, characteristics, and practical applications. To close the knowledge gap between theory and practice, the project also investigates the practical application of spherical fuzzy hypersoft sets to real-world problems.

1.3 | Main Structure

The sections that follow are organized as follows:

- The fundamental definitions needed to comprehend spherical fuzzy hypersoft sets are provided in Section 3. This chapter first lays the groundwork by elucidating the basic concepts of soft sets, fuzzy sets, and hypersoft sets in order to synthesize these notions into spherical fuzzy hypersoft sets.

- Section 4 delves into the basic operations of spherical fuzzy hypersoft sets and looks closely at how they are created. The mechanics of spherical fuzzy hypersoft sets are thoroughly explained in this chapter through an examination of parameterization, membership assignment degree, and set operations such as complement, union, and intersection.
- Section 5 explains the suggested technique's methodology.
- In Section 6, the focus is redirected to practical applications. Through practical case examples, it illustrates the adaptability of spherical fuzzy hypersoft sets. This chapter describes applications of spherical fuzzy hypersoft sets to image processing techniques and healthcare diagnostic systems that demonstrate their efficacy in solving challenging problems.
- Section 7 presents the results of the study's exploration of spherical fuzzy hypersoft sets. This chapter proposes future paths and research issues while highlighting the continued importance and promise of spherical fuzzy hypersoft sets.

By giving a comprehensive analysis of the concept of spherical fuzzy hypersoft sets and offering a helpful resource for both scholars and decision-makers, this work seeks to contribute to the expanding field of uncertainty modeling. We are about to embark on a journey that, by embracing the merger of soft sets, fuzzy sets, and parameterized uncertainty, could drastically change the way we approach and resolve difficult problems in an uncertain world.

2 | Literature Review

Since fuzzy set theory addresses the challenges of obtaining enough data for useful decision-making due to uncertainty in socioeconomic elements and data availability, it is frequently utilized as an effective solution to problems with multi-attribute decision-making procedures. The inventor of fuzzy sets was Lotfi Zadeh [10] in 1965 a method for dealing with vague and non-specific information. When working with data that is difficult to quantify or lacks distinct category boundaries, they are especially helpful. A fuzzy set's partial membership assigns a number between 0 and 1 to each element, denoting that element's membership level in the set. The membership function within the set controls how much the set contains each element, permitting a nuanced depiction of uncertainty. Complete membership is represented by a value of one, whereas non-membership is indicated by a value of 0. Degrees of fuzziness or partial membership are represented by values between 0 and 1. Artificial intelligence, control systems, data mining, pattern recognition, and other fields frequently use fuzzy sets. They offer a potent tool for managing and making sense of imprecise and uncertain data, enabling more adaptable and realistic modeling.

In circumstances where it is challenging to define precise rules, fuzzy logic expands classical binary logic to handle fuzzy sets frequently used in conjunction with fuzzy sets to perform reasoning and make judgments. Approximate reasoning can be represented using fuzzy logic, which also effectively represents the ambiguity and uncertainty prevalent in many real-world problems that call for creative solutions. Using type 2 fuzzy sets, Zadeh (1975) developed the idea of building upon classical fuzzy sets. Type-2 fuzzy sets provide a more thorough framework for capturing uncertainty or ambiguity than standard fuzzy sets provide. Fuzzy type-2 sets have a membership function that is not limited to a single value but rather includes a range of possible values that together represent the degree of uncertainty associated with each membership level. The membership function in type-2 fuzzy sets shows two levels of uncertainty. Similar to conventional fuzzy sets, the first layer relates to the degree of inclusion within the set. The next layer, which is a range of values normally limited by higher and lower membership grades, symbolizes doubt regarding the membership degree. The unique benefit of type-2 fuzzy sets is that they can manage ambiguity and uncertainty better than type-1 fuzzy sets. They can simulate scenarios in which precise knowledge is either lacking or inconsistent in the available data. Because type-2 fuzzy sets capture the uncertainty associated with membership degrees, they are capable of handling a wider range of real-world problems. However, due to the additional procedures required to handle membership grade uncertainty, type-2 fuzzy sets are more computationally complex than

type-1 fuzzy sets. Many techniques and algorithms, such as interval arithmetic, interval-type-2 fuzzy sets, and type-reduction techniques, have been developed to handle type-2 fuzzy sets.

Type-2 fuzzy sets are used in a variety of domains, such as data mining, pattern recognition, control systems, and decision-making, to address situations that call for skill in managing ambiguity and uncertainty. They provide a more robust and flexible framework for reasoning and modeling in complex and unpredictable contexts. An extension of classical fuzzy sets, intuitionistic fuzzy sets (IFS) provides more flexible representation options beyond the basic membership of uncertainty and ambiguity. Introduced by Atanassov [11] in 1986, IFS offers a framework for managing inaccurate or partial data by adding extra layers of In traditional fuzzy set theory, the degree of an element's membership in a set is represented by its membership degree, which ranges from 0 to 1. On the other hand, intuitionistic fuzzy sets take this idea a step further by including non-membership and reluctance. While reluctance describes the degree of doubt or unfamiliarity with an element's membership status, non-membership reflects an element's degree of exclusion from the set. Speaking now on spherical fuzzy soft sets: [12] creating a more flexible membership function configuration for the increased uncertainty framework. Shape takes on a significant role. Utilizing fuzzy numbers [13], They make it possible to depict a range of possible membership degrees rather than just one, skillfully encapsulating the imprecision prevalent in real-world situations. Intuitionistic fuzzy sets present a comprehensive and flexible method for representing ambiguity and uncertainty in real-world scenarios. They perform exceptionally well in situations requiring high levels of non-membership, which is not a feature of traditional fuzzy sets. Fuzzy intuition has proven useful in many fields, including expert systems, control systems, image processing, pattern identification, and decision-making. Atanassov [14] established operations on fuzzy sets [15] to aid in inference and judgment, including union, complement, and other operations inside intuitionistic fuzzy sets. There are developed intersection and arithmetic procedures. These procedures take into account the three factors that characterize intuitionistic fuzzy sets, producing results that reflect the underlying ambiguity and reluctance in the dataset.

In an extended sense, intuitionistic fuzzy sets generalize the basic concepts of fuzzy sets by including the idea of reluctance or non-membership. This enhanced framework offers a more comprehensive structure for handling ambiguity and imprecision in a variety of applications, facilitating a more realistic representation of complex and ambiguous data. Certain medical situations have seen the practical application of intuitionistic fuzzy sets [16]. In 2007, the idea of fuzzy multiset was first proposed [17] and used in many fields [18-20]. Neutrosophic sets [21], also known as neutrosophy or neutrosophic logic, represent an improvement on fuzzy sets. that permits the representation and handling of indeterminacy, ambiguity, and inconsistency in a more nuanced way. Neutrosophy was introduced by the philosopher and mathematician Florentin Smarandache [22] in the 1998s. Florentin Smarandache [23] presented the extension of soft sets to hypersoft sets. He expanded by converting the function F into a multi-attribute function, one can turn a soft set into a hypersoft set, yielding a hypersoft set in the process parameterized collection of soft sets. This advanced structure goes beyond being a mere assembly of sets, presenting a collection of collections of sets. Hypersoft sets have found applications in decision-making, data mining, image processing, and addressing uncertainty in natural language processing and artificial intelligence. Within the hypersoft set framework, fundamental properties like non-Set, Subset, Absolute Set, and Aggregation Operations such as Restricted Union, Extended Intersection, Relevant Complement, Restricted Difference, and Restricted Symmetric Difference are defined, supplemented by illustrative examples. Novel notions of connection, function, and fundamental attributes are introduced "Regarding hypersoft sets, an illustrative display includes a matrix portrayal, accompanied by an array of operations." in [24]. Basic operations on soft sets were also discussed in [25] in detail. Detailed study of hypersoft sets were discussed in [26-28]. Florentin Smarandache [23] "defined as a fusion of the foundational concepts of fuzzy sets and hypersoft sets, we have crisp hypersoft sets. These sets incorporate the fundamental principles from both fuzzy sets and hypersoft sets to create a novel framework known as fuzzy hypersoft sets" which offer an enhanced framework to effectively handle uncertainty, vagueness, and indeterminacy present within data. They allow for the representation of complex membership structures that can capture various degrees of uncertainty in a more nuanced way. In fuzzy hypersoft sets [23], the level of

an element's membership signifies its extent of affiliation with a specific set, whereas the indeterminacy degree quantifies the uncertainty linked to that membership degree. This indeterminacy degree imparts supplementary insights into the level of uncertainty and ambiguity surrounding the membership. of ambiguity or lack of information regarding the membership status of an element. This concept was later used in many fields [22, 28-32]. In Pythagorean fuzzy sets [33], each element has three values associated with it: comprising these three factors that describe the nature of items within a set: a level of mem, a level of Nmem, and a level of indeterminacy. The level of indeterminacy indicates the level of ambiguity or uncertainty around both the level of mem and the level of Nmem aspects. The level of meme measures the level of an element's affiliation, and the degree of non-membership measures its non-affiliation. Further properties and operations were discussed in [34-37]. Membership and non-membership levels, which indicate partial or complete absence of membership, are established within the range of $[0,1]$. Like that, the indeterminacy degree, which also spans $[0, 1]$, denotes whether there is some indeterminacy or all of it. Picture fuzzy sets (PFS) are an expansion of both fuzzy sets and intuitionistic fuzzy sets. Kreinovich and Cuong invented it [38]. A PFS is defined as a mapping from a discourse domain X to a set with three membership degrees-positive, negative, and neutral degrees sum up to 1.

PFSs can be used [39] to model uncertainty in situations where there are more than two possible answers. For example, a PFS could be used to model the results of a survey where participants were requested to evaluate a product using a scale ranging from 1 to 5. The positive membership degree would represent the number of respondents who gave the product a rating of 4 or 5, and the number of negative members would equal the level of negative mem. The percentage of respondents giving a product a rating of 1 or 2 coupled with the percentage of neutral respondents equals the number of respondents giving the product a rating of 3. Whenever making decisions, Picture Fuzzy Sets (PFSs) prove valuable for representing varied viewpoints of decision-makers. For instance, a PFS might depict diverse investor opinions within a group considering an investment in a specific stock. Detailed discourse concerning picture fuzzy sets and the operations performed on fuzzy sets are as follows and are discussed in detail in [40-42]. The notion of a Picture fuzzy hypersoft set was invented in [43]. Picture fuzzy soft set builds upon the foundation laid by the idea of picture fuzzy soft sets. A detailed discussion on picture fuzzy hypersoft sets is done in detail, with operations of union and intersection. The definition of extended union and restricted intersections are defined and other properties of with applications are discussed in [43-44]. Its membership function defines a spherical fuzzy set, which establishes a correlation between points on a sphere and real numbers that fall within the range of $[0,1]$. This membership function quantifies the extent to which a point on the sphere pertains to the set. The definition of spherical fuzzy numbers, their inherent properties, and the establishment of the union and intersection operations are outlined within the context. [45]. The extension of the TOPSIS multi-criteria decision-making approach is converted into spherical fuzzy TOPSIS, and an exemplary case is shown. In addition, a comparison with intuitionistic fuzzy TOPSIS (IF-TOPSIS) is performed, and other concepts and their use in decision-making are discussed in [46].

3 | Preliminaries

Definition 1. (Fuzzy Set)

In the context of a fuzzy set [10] theory, denoted as A, operating within the universe of discourse $\mho'$, its definition is reliant on a membership function $\mu_A(\varrho')$. This function allocates a membership degree to every element ϱ' present in $\mho'$. The membership function, $\mu_A(\varrho')$, essentially transforms each component into a level of membership to a number in the range $[0,1]$, showing how closely ϱ' is related to A.

The membership function can be expressed mathematically as follows:

$$\mu_A(\varrho'): \mho' \to [0,1]$$
$$A = \{(\varrho', \mu_A(\varrho')): \varrho' \in \mho'\}$$

When performing the union operation on fuzzy sets, care must be made to account for the membership values assigned to each element within the sets. The combination of two fuzzy sets, represented as A and B, is $A \cup B$, yielding a unique fuzzy set. Each element's membership value in this union is equal to the maximum of its corresponding membership values in sets A and B. It is also possible to expand this union strategy to include more than two sets. When three fuzzy sets A, B, and C are taken into consideration, for example, their union would be written as $A \cup B \cup C$.

If A is defined as $(\varrho', \mu_A(\varrho'))$ and B as $(\varrho', \mu_B(\varrho'))$ for the same universe of discourse $\mho'$, their union denoted by $C = A \cup B$ can be formulated as:

$$C = \left\{\left(\varrho', \max\left(\mu_A(\varrho'), \mu_B(\varrho')\right)\right)\right\}$$

The intersection of fuzzy sets is a technique for merging two or more fuzzy sets into a single new fuzzy set. An element's membership degree within the resultant fuzzy set is determined by the minimum of its membership degrees across the underlying fuzzy sets. This intersection function is crucial to fuzzy logic since uncertainties and partial memberships are common in pattern recognition, control systems, and decision-making.

A mathematical process known as each element in a fuzzy set has a level of Nmem that is determined by the complement of the fuzzy set. It quantifies the extent to which an element is not considered part of the set. It's important to distinguish between the complement of a crisp set and that of a fuzzy set. Unlike the complement of a crisp set, which encompasses all elements the complement is absent from the set in fuzzy sets signifies the degree of non-membership. This concept goes beyond simple binary classification of whether an element belongs or does not belong.

Definition 2. (Intuitionistic Fuzzy Sets) [11]

An Intuitionistic Fuzzy set W stated as

$$W = \{(c', \langle A_W(c'), B_W(c')\rangle) \mid c' \in M\}$$

such that $A_W: M \to I$ and $B_W: M \to I$, where $A_W(c')$ and $B_W(c')$ specify the value of belonging and the value of not belonging to $c' \in W$ with the restriction that $0 \leq A_W(c') + B_W(c') \leq 1$ and the degree of hesitancy $H_W(c') = 1 - A_W(c') - B_W(c')$.

Definition 3. (Soft Set)

If X is the universal set and E is a set of parameters, $P(X)$ represents the power set of X. Let A be a set of effective parameters where $A \subset E$. The concept of a soft set, as developed by Moldsove [47], is defined by the mapping.

$$W: A \to P(X)$$

The above definition shows the soft set Commencing from A, which constitutes a set of effective attributes, and relating it to the power set of X (where X represents a universal set), a soft set emerges as a mathematical entity defined by: the framework that expands on the idea of a set by allowing elements to have degrees of membership rather than the traditional binary notion of belonging or not belonging. In a soft set, each element is assigned a membership function that determines its degree of inclusion in the set. Soft sets allow for a more flexible representation of uncertainty and ambiguity compared to traditional crisp sets. They find applications in a multitude of fields, spanning decision-making and pattern recognition data analysis, and information fusion, where imprecise or uncertain information needs to be handled.

Definition 4. (Spherical Fuzzy Sets) [45]

A spherical fuzzy set A in a universe of discourse X is defined by its core $c_A \in X$, radius $r_A \geq 0$, and membership function $\mu_A: X \to [0,1]$ such that for any element x in X :

$$\mu_A(x) = \begin{cases} 1 - \dfrac{d(x, c_A)}{r_A}, & \text{if } d(x, c_A) \leq r_A \\ 0, & \text{otherwise} \end{cases}$$

where $d(x, c_A)$ represents the distance between x and the core c_A.

Definition 5. (Fuzzy Soft Set) [48]

A fuzzy soft set FS in a universe of discourse X is defined by a triple $(\tilde{E}, \lambda, \pi)$, where:

- $\tilde{E}$ is the underlying set of elements with a membership function $\mu_{\tilde{E}}: X \to [0,1]$ representing the level of inclusion of components in $\tilde{E}$.
- λ is the parameter set associated with $\tilde{E}$.
- $\pi: \tilde{E} \times \lambda \to [0,1]$ gives each constituent a membership degree of $\tilde{E}$ with respect to each parameter in λ.

For any element x in $\tilde{E}$ and parameter p in λ, the membership degree of x with respect to p is denoted as $\pi(x, p)$.

Fuzzy soft sets amalgamate characteristics from both fuzzy sets and soft sets, providing a framework to concurrently handle data that is unclear and ambiguous. Within a fuzzy soft set, elements possess varying degrees of membership while being linked to diverse potential values, thereby capturing uncertain or imprecise information regarding the elements.

Definition 6. (Hypersoft Sets)

The idea of a hypersoft set, developed by Smarandache in 2018 [23], involves a pair denoted as (ζ, G), which operates within the universe U. In this framework, G represents the Cartesian product of n separate attribute-valued sets, denoted as $G_1, G_2, \dots, G_n$. These sets correspond to distinct properties $g_1, g_2, \dots, g_n$ respectively, and they are defined as follows:

$$\zeta: G \to \mathcal{P}(U)$$

The mapping ζ establishes the essential foundation of a hypersoft set, associating elements from G with subsets of U.

Definition 7. (Fuzzy Hypersoft Sets)

Suppose $F(U)$ is the collection of fuzzy sets over U. Let $\{a_1, a_2, a_3, \dots, a_n\}$ be different properties, and their corresponding value sets be $G_1, G_2, G_3, \dots, G_n$. A fuzzy hypersoft set [49] is known as $(\zeta_{\text{fhs}}, G) = \{(g, \zeta_{\text{fhs}}(g)): g \in G, \zeta_{\text{fhs}}(g) \in F(U)\}$, where $\zeta_{\text{fhs}}: G \to F(U)$, and $\forall g \in G$,

$$G = G_1 \times G_2 \times G_3 \times \dots \times G_n$$

The membership function $\zeta_{\text{fhs}}(g)$ for each property combination, g is described as follows:

$$\zeta_{\text{fhs}}(g) = \{\mu\phi_{\text{fhs}}(g)/u: u \in U, \mu\phi_{\text{fhs}}(g(u)) \in [0,1]\}$$

Here, $\mu\phi_{\text{fhs}}(g)$ represents the level of membership for the property combination $g(u)$ of element u in the universe of discourse U, with the membership value within the interval $[0,1]$.

The fuzzy hypersoft set framework captures relationships between properties and their values using fuzzy sets, allowing for a flexible representation of uncertainty and imprecision in a structured manner.

Definition 8. (Intuitionistic Fuzzy Hypersoft Sets)

Assume U is a universal set. An intuitionistic fuzzy hypersoft set [29] (IFHs-set) over U is represented by a mapping $A: U \to [0,1] \times [0,1]^c, c$ is a positive integer, in this case. In this mapping, for each element $x \in U$, there exists a pair of values (μ, λ), where μ and λ belong to the intervals $[0,1]$ and $[0,1]^c$, respectively. These values μ and λ quantify the level of mem and Nmem, respectively, for the element x within the set.

In simpler terms, an intuitionistic fuzzy hypersoft set characterizes each element in the universal set by indicating the strength of its belongingness (level of mem) and the strength of its non-belongingness (degree of non-membership) to the set. This representation allows for more intricate handling of uncertainty and imprecision.

Definition 9. (Interval-Valued Intuitionistic Fuzzy Set)

Suppose that $\mho$ is a universe of discourse then an IVIFS [50] can be described as:

$$F = \{\langle \varrho, \mu_F(\varrho), V_F(\varrho) \rangle \mid \varrho \in \mho\}$$

where ϱ is the set of elements of $\mho. \mu_F(\varrho)$ represents the interval-valued degree membership of $\mho$ as $[\mu_F^+, \mu_F^-] \subseteq [0,1]$ and $V_F(\varrho)$ represents the interval-valued degree non-membership of all elements of $\mho$ as $[V_F^+, V_F^-] \subseteq [0,1]$. The sum of the supremum of level membership and supremum of level non-membership lies between 0 and 1 as

$$0 \leq \sup\mu_F(\varrho) + \sup V_F(\varrho) \leq 1,\ \varrho \in \mho$$

The provided diagram illustrates that Crisp sets, Fuzzy sets, and Intuitionistic Fuzzy sets (IFS) are subsets of Interval Valued Intuitionistic Fuzzy sets (IVIFS). Here, π_F (varrho) denotes the level of N-determinacy within set mem. When both the lower and upper bounds of degree membership and degree Nmem align, an IVIFS transforms into an IFS. A crisp set adheres to the traditional set concept where elements are either included or excluded. Each element possesses a definite membership value of either 0 or 1. Fuzzy sets accommodate varying degrees of membership, reflecting uncertainty or ambiguity. Elements within a fuzzy set are assigned membership values ranging from 0 to 1, signifying their extent of belonging to the set. Rather than assigning a single membership value to each element, IVFS assigns a range of values that represents the degree of membership. This interval may vary from a single point to a wide range of values.

Definition 10. (Spherical Fuzzy Soft Sets)

Assume the universe set be denoted as $\mathbb{R}$. To explain the idea of a Spherical Fuzzy Soft Set [12], we can represent it as follows:

$$A^* = \{\langle r, P_A(r), I_A(r), N_A(r) \rangle : r \in \mathbb{R}\}$$

where:

$$P_A(r): \mathbb{R} \to [0,1]$$
$$I_A(r): \mathbb{R} \to [0,1]$$
$$N_A(r): \mathbb{R} \to [0,1]$$

In this context:

- $P_A(r)$ signifies the level of favorable membership of element r in the set $\mathbb{R}$.
- $I_A(r)$ represents the level of neutral membership of element r in $\mathbb{R}$.
- $N_A(r)$ indicates the level of negative membership of element r in $\mathbb{R}$.

It's important to satisfy the following conditions for these membership functions:

$$0 \leq P_A^2(r) + I_A^2(r) + N_A^2(r) \leq 1$$

This formulation captures the essence of a Spherical Fuzzy Soft Set, wherein elements in the universe $\mathbb{R}$ are characterized based on their degrees of membership that are favorable, neutral, and negative while ensuring that the sum of squared membership degrees remains bounded within the interval $[0,1]$.

Fuzzy values assigned to points in a multidimensional space provide membership degrees for spherical fuzzy sets, which are a subset of classical fuzzy sets. In contrast to conventional fuzzy sets predominantly delineated along linear scales, spherical fuzzy sets manifest on a spherical continuum. Within a spherical fuzzy set, every point within the multidimensional space links to a fuzzy value signifying its membership degree. These fuzzy values are often visualized as points on a unit sphere, where the distance from the sphere's center symbolizes the extent of membership.

Definition 11. (score and accuracy function)

Suppose $\beta = (\varrho_\beta, \varphi_\beta)$ be a SFHSS.

- The mathematical representation of a score function is typically denoted as $\mathbb{S}(\beta) = \varrho_\beta^2 - \varsigma_\beta^2 - \varphi_\beta^2$.
- The mathematical representation of an accuracy function is typically denoted as $\mathbb{A}(\beta) = \varrho_\beta^2 + \varsigma_\beta^2 + \varphi_\beta^2$.

4 | Spherical Fuzzy Hypersoft Set

This section introduces the idea of the spherical fuzzy hypersoft set (SFHS) and its operations. It will cover the values between 0 and 1, but the advantages here are that in a hypersoft set, we can take multi-attribute values instead of single-attribute values. In other words, we take attributes and their corresponding attribute values instead of single attribute values, like we take single values in soft sets. We will talk about some fundamental manipulations on spherical fuzzy hypersoft sets in this part. The Subset of two spherical fuzzy hypersoft numbers is known and further created with the help of numerical examples. An extended Union of two spherical fuzzy hypersoft numbers is defined, as an extended intersection, union with restrictions Restricted intersections, and complement of spherical fuzzy hypersoft numbers. In the next part of this section, further operations of addition and multiplications of spherical fuzzy hypersoft numbers are defined. Basic properties of spherical fuzzy hypersoft numbers like commutative property w.r.t addition and multiplication are defined. The associative property of spherical fuzzy hypersoft numbers is defined w.r.t addition and multiplication. In the end, some other properties are also defined.

Definition 12. Spherical Fuzzy Hypersoft Set

Let U be a universe of discourse. Let E be a set of distinct attributes and $A \subset E$ a pair $\langle G, A^* \rangle$.

$$G: A^* \to SFHS(U)$$

or

$$G: A^* \to P(U)$$

where A^* is the Cartesian product of n disjointly valued sub-parameter sets $\{A_1, A_2, \dots, A_n\}$,

$$n \geq 1$$

corresponding to distinct parameters $e_1, e_2, \dots, e_n$ respectively. e^* of A^* is an n-tuple of elements.

$$G(e^*) = \langle x, P_{G(e^*)}(x), N_{G(e^*)}(x), I_{G(e^*)}(x) \rangle : x \in U$$

where $P_{G(e^*)}(x)$ is called the level of favorable membership, $I_{G(e^*)}(x)$ is called the level of non-favorable membership, and $N_{G(e^*)}(x)$ is called the degree of neutral membership.

$P_{G(e^*)}(x), I_{G(e^*)}(x)$, and $N_{G(e^*)}(x)$ satisfy the following conditions:

$$0 \leq P^2_{G(e^*)}(x) + N^2_{G(e^*)}(x) + I^2_{G(e^*)}(x) \leq 1$$

and

$$\psi_{sfhs}(g) = \{\mu\psi_{sfhs}(g)(x), \eta\psi_{sfhs}(g)(x), \nu\psi_{sfhs}(g)(x)\}$$
$$\mu\psi_{sfhs}(g)(x) \in [0,1]$$
$$\eta\psi_{sfhs}(g)(x) \in [0,1]$$
$$\nu\psi_{sfhs}(g)(x) \in [0,1]$$

for a spherical fuzzy hypersoft set

$$\{x, P_{G(e^*)}(x), N_{G(e^*)}(x), I_{G(e^*)}(x): x \in U\}$$

which is called a triple component.

$$\langle P_{G(e^*)}(x), N_{G(e^*)}(x), I_{G(e^*)}(x)\rangle$$

are called spherical fuzzy hypersoft numbers (SFHSNs). can be denoted by

$$r = \langle P_r, N_r, I_r\rangle$$

where $P_r \in [0,1]$, $N_r \in [0,1]$, $I_r \in [0,1]$, with the condition

$$0 \leq P_r^2 + N_r^2 + I_r^2 \leq 1$$

Definition 13. (Averaging and geometric operator)

Suppose $\beta_1, \dots, \beta_n$ be some collection of SFHSNs as The SFHSA operator is called $\mathrm{SFHSA}(\beta_1, \dots, \beta_n)$

$$\left(\sqrt[n]{\left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\varrho_{\beta_i})^n\right)\right)\right)}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\varsigma_{\beta_i})^n\right), \prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\varphi_{\beta_i})^n\right)\right) \tag{1}$$

The SFHSG operator is called $\mathrm{SFHSG}(\beta_1, \dots, \beta_n) =$

$$\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(\varrho_{\beta_i})^n\right), \sqrt[n]{\left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\varsigma_{\beta_i})^n\right)\right)\right)}, \sqrt[n]{\left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - (\varphi_{\beta_i})^n\right)\right)\right)}\right) \tag{2}$$

4.1 | Some Fundamental Operations with Spherical Fuzzy Hypersoft Sets

Assuming the two spherical fuzzy hypersoft numbers

$$r_j = \langle P_{rj}, N_{rj}, I_{rj}\rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk}\rangle$$

be two SFHS.

4.2 | Subset

The subset of the two numbers

$$r_j = \langle P_{rj}, N_{rj}, I_{rj}\rangle$$

and

$$r_k = \langle P_{rk}, N_{rk}, I_{rk}\rangle$$

is defined as $r_j \subset r_k$ if for all $x \in U$,

$$P_{rj} \leq P_{rk}$$
$$N_{rj} \leq N_{rk}$$
$$I_{rj} \geq I_{rk}$$

4.3 | Numerical Example

Consider that a person wishes to select a hospital for treatment. A number of three hospitals which is a set of discourse $A \subset E$.

$$U = \{Z_1, Z_2, Z_3, Z_4\}$$

Attributes and their values are:

- $v_1 =$ Location
- $v_2 =$ All Facilities
- $v_3 =$ Treatment Fee

Then the attribute-valued sets are:

$$\text{Location } = V_1 = \{v_{11} = \text{ near}, v_{12} = \text{ far}\}$$
$$\text{All Facilities } = V_2 = \{v_{21} = \text{ available}, v_{22} = \text{ not available}\}$$
$$\text{Treatment Fee } = V_3 = \{v_{31} = \text{ cheap}\}$$
$$Y = V_1 \times V_2 \times V_3$$
$$Y = \{y_1, y_2, y_3, y_4\}$$

where

$$y_1 = \{v_{11}, v_{21}, v_{31}\}$$
$$y_2 = \{v_{11}, v_{22}, v_{31}\}$$
$$y_3 = \{v_{12}, v_{21}, v_{31}\}$$
$$y_4 = \{v_{12}, v_{22}, v_{31}\}$$
$$A = \{(\zeta_{\text{fhss}}, y_1) = (0.3,0.1,0.3)/Z_1, (0.4,0.1,0.3)/Z_2, (0.2,0.3,0.4)/Z_3, (0.5,0.1,0.3)/Z_4,$$
$$(\zeta_{\text{fhss}}, y_2) = (0.2,0.1,0.3)/Z_1, (0.3,0.2,0.3)/Z_2, (0.2,0.3,0.4)/Z_3, (0.4,0.1,0.3)/Z_4$$
$$(\zeta_{\text{fhss}}, y_3) = (0.2,0.1,0.4)/Z_1, (0.3,0.1,0.3)/Z_2, (0.3,0.1,0.4)/Z_3, (0.3,0.1,0.3)/Z_4,$$
$$(\zeta_{\text{fhss}}, y_4) = (0.4,0.1,0.3)/Z_1, (0.5,0.2,0.3)/Z_2, (0.2,0.1,0.4)/Z_3, (0.5,0.2,0.3)/Z_4\}$$
$$V = \{(\zeta_{\text{fhss}}, y_1) = (0.4,0.3,0.2)/Z_1, (0.5,0.2,0.2)/Z_2, (0.3,0.4,0.3)/Z_3, (0.6,0.3,0.2)/Z_4,$$
$$(\zeta_{\text{fhss}}, y_2) = (0.5,0.2,0.1)/Z_1, (0.4,0.3,0.1)/Z_2, (0.3,0.4,0.1)/Z_3, (0.6,0.3,0.2)/Z_4,$$
$$(\zeta_{\text{fhss}}, y_3) = (0.4,0.3,0.2)/Z_1, (0.5,0.2,0.1)/Z_2, (0.4,0.3,0.3)/Z_3, (0.4,0.3,0.2)/Z_4,$$
$$(\zeta_{\text{fhss}}, y_4) = (0.5,0.3,0.2)/Z_1, (0.6,0.3,0.2)/Z_2, (0.4,0.3,0.3)/Z_3, (0.6,0.3,0.2)/Z_4\}$$

Then $A \subseteq V$.

4.4 | Extended Union

Assuming the two spherical fuzzy hypersoft numbers

Let $r_j =< P_{rj}, N_{rj}, I_{rj} >$ and $r_k =< P_{rk}, N_{rk}, I_{rk} >$ be two SFHSs.

The extended union of the two numbers

$$r_j =< P_{rj}, N_{rj}, I_{rj} >$$

and

$$r_k =< P_{rk}, N_{rk}, I_{rk} >$$

is defined as

$$G(e^*) = \{< x, \max(P_{(rj)}, P_{(rk)}), \min(N_{(rj)}, N_{(rk)}), \min(I_{(rj)}, I_{(rk)}) >: x \in X\}$$

where $G(e^*)$ denotes the extended union of two SFHSs.

4.5 | Numerical Example

Let $X = \{x_1, x_2, x_3\}$ be the universe of discourse.

Set :

- Center (0.6,0.7,0.8)
- Radius 0.9
- Membership Function $\mu_A(x)$

Set :

- Center (0.2,0.3,0.4)
- Radius 0.5
- Membership Function $\mu_B(x)$

The extended union of sets A and B is given by:

$$\mu_{A \oplus B}(x) = \max\left(\min\left(\frac{\| x - (0.6,0.7,0.8) \|}{0.9}, 1\right), \min\left(\frac{\| x - (0.2,0.3,0.4) \|}{0.5}, 1\right)\right)$$

where $\| x - c \|$ represents the Euclidean distance between point x and the center c.

4.6 | Extended Intersection

The Extended Intersection of the two numbers

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle,\ r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$

The intersection $r_j \cap r_k$ is defined as:

$$r_j \cap r_k = \langle \min(P_{rj}, P_{rk}), \min(N_{rj}, N_{rk}), \max(I_{rj}, I_{rk}) \rangle$$

4.7 | Numerical Example

Let $X = \{x_1, x_2, x_3\}$ be the universe of discourse.

Set :

- Center (0.6,0.7,0.8)
- Radius 0.9
- Membership Function $\mu_A(x)$

Set :

- Center (0.2,0.3,0.4)
- Radius 0.5
- Membership Function $\mu_B(x)$

The extended intersection of sets A and B is given by:

$$\mu_{A\ominus B}(x) = \max\left(0, \min\left(\frac{\| x - (0.6,0.7,0.8) \|}{0.9}, \frac{\| x - (0.2,0.3,0.4) \|}{0.5}\right)\right)$$

where $\| x - c \|$ represents the Euclidean distance between point x and the center c.

5 | Restricted Union

Assuming the two spherical fuzzy hypersoft numbers

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$

are two SFHSs, the Restricted Union of r_j and r_k is defined as

$$G(e^*) = \{\langle x, \max(P_{(rj)}, P_{(rk)}), \max(N_{(rj)}, N_{(rk)}), \min(I_{(rj)}, I_{(rk)}) \rangle : x \in X\}$$

where $G(e^*)$ denotes the restricted union of two SFHSS. The intersection of r_j and r_k is non-empty.

6 | Numerical Example

Let $X = \{x_1, x_2, x_3\}$ be the universe of discourse.

Set: Center $(0.6,0.7,0.8)$

Radius 0.9

Membership Function $\mu_A(x)$

Set: Center $(0.2,0.3,0.4)$

Radius 0.5

Membership Function $\mu_B(x)$

The restricted union of sets A and B is given by:

$$\mu_{A\otimes B}(x) = \begin{cases} \min\left(1, \max\left(\frac{\| x - (0.6,0.7,0.8) \|}{0.9}, \frac{\| x - (0.2,0.3,0.4) \|}{0.5}\right)\right) & \text{if } \max(\mu_A(x), \mu_B(x)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\| x - c \|$ represents the Euclidean distance between point x and the center c.

7 | Restricted Intersection

Assuming the two spherical fuzzy hypersoft numbers

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$

are two SFHSs, the Restricted Intersection of r_j and r_k is defined as

$$G(e^*) = \{\langle x, \max(P_{(rj)}, P_{(rk)}), \max(N_{(rj)}, N_{(rk)}), \min(I_{(rj)}, I_{(rk)}) \rangle : x \in X\}$$

where $G(e^*)$ denotes the restricted intersection of two SFHSS. The restricted intersection of r_j and r_k is non-empty.

8 | Numerical Example

Let $X = \{x_1, x_2, x_3\}$ be the universe of discourse.

Set: Center $(0.6,0.7,0.8)$

Radius 0.9

Membership Function $\mu_A(x)$

Set: Center $(0.2,0.3,0.4)$

Radius 0.5

Membership Function $\mu_B(x)$

The restricted intersection of sets A and B is given by:

$$\mu_{A\odot B}(x) = \begin{cases} \max\left(0, \min\left(\frac{\| x - (0.6,0.7,0.8) \|}{0.9}, \frac{\| x - (0.2,0.3,0.4) \|}{0.5}\right)\right) & \text{if } \min(\mu_A(x), \mu_B(x)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\| x - c \|$ represents the Euclidean distance between point x and the center c.

9 | Complement

Assuming the spherical fuzzy hypersoft number

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$

the Complement of r_j is defined as

$$r_j^c = \langle I_{rj}, N_{rj}, P_{rj} \rangle$$

10 | Numerical Example

Let $X = \{x_1, x_2, x_3\}$ be the universe of discourse.

Set: Center $(0.6,0.7,0.8)$

Radius 0.9

Membership Function $\mu_A(x)$

The complement of set A is given by:

$$\mu_{\bar{A}}(x) = 1 - \min\left(1, \frac{\| x - (0.6,0.7,0.8) \|}{0.9}\right)$$

where $\| x - c \|$ represents the Euclidean distance between point x and the center c.

10.1 | Operations on Spherical Fuzzy hypersoft Numbers

Suppose two SFHSs

$$A = r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$B = r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$

and $\tau \geq 0$, the operations of SFHSNs can be defined as follows:

$$\tau r_j = \left\langle \sqrt{1-\left(1-P_{rj}\right)^{\tau}}, \left(N_{rj}\right)^{\tau}, \left(I_{rj}\right)^{\tau} \right\rangle \tag{1}$$

$$r_j + r_k = \left\langle \sqrt{\left(P_{rj}\right)^2 + (P_{rk})^2 - \left(P_{rj}\right)^2 \times (P_{rk})^2}, N_{rj} \times N_{rk}, I_{rj} \times I_{rk} \right\rangle \tag{2}$$

$$r_j \times r_k = \left\langle P_{rj} \times P_{rk}, N_{rj} \times N_{rk}, \sqrt{\left(I_{rj}\right)^2 + (I_{rk})^2 - \left(I_{rj}\right)^2 \times (I_{rk})^2} \right\rangle \tag{3}$$

$$\left(r_j\right)^{\tau} = \left\langle \left(P_{rj}\right)^{\tau}, \left(N_{rj}\right)^{\tau}, \sqrt{1-\left(1-I_{rj}\right)^{\tau}} \right\rangle \tag{4}$$

Theorem 1. Assuming three SFHSS numbers as

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$r_l = \langle P_{rl}, N_{rl}, I_{rl} \rangle$$

and

$$\tau \geq 0$$
$$\tau_j \geq 0$$
$$\tau_k \geq 0$$

then the enumerated identities are met.

i). $r_j + r_k = r_k + r_j$

Commutative property w.r.t. addition

ii). $r_j \times r_k = r_k \times r_j$

Commutative property w.r.t. multiplication

iii). $\left(r_j + r_k\right) + r_l = r_j + (r_k + r_l)$

Associative property w.r.t. addition

iv). $\left(r_j \times r_k\right) \times r_l = r_j \times (r_k \times r_l)$

Associative property w.r.t. multiplication

v). $\left(\tau r_j + \tau r_k\right) = \tau\left(r_j + r_k\right)$

Left distributive property

vi). $\left(\tau_j r_j + \tau_k r_k\right) = \left(\tau_j + \tau_k\right) r_j$

Right distributive property

vii). $\left(\tau_j r_j + \tau_k r_k\right) = \left(\tau_j + \tau_k\right) r_j$

Right distributive property

viii). $\left(\left(r_j\right)^{\tau_j} + \left(r_j\right)^{\tau_k}\right) = \left(r_j\right)^{\left(\tau_j + \tau_k\right)}$

Left distributive property

Proof. (i)

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$r_l = \langle P_{rl}, N_{rl}, I_{rl} \rangle$$
$$\tau \geq 0$$

To show that

$$r_j + r_k = r_k + r_j$$

L.H.S =

$$r_j + r_k$$
$$\langle P_{rj}, N_{rj}, I_{rj} \rangle + \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$= \left\langle \sqrt{(P_{rj})^2 + (P_{rk})^2 - (P_{rj})^2 \times (P_{rk})^2}, N_{rj} \times N_{rk}, I_{rj} \times I_{rk} \right\rangle$$
$$= \left\langle \sqrt{(P_{rk})^2 + (P_{rj})^2 - (P_{rk})^2 \times (P_{rj})^2}, N_{rk} \times N_{rj}, I_{rk} \times I_{rj} \right\rangle$$
$$= \langle P_{rk}, N_{rk}, I_{rk} \rangle + \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$= r_k + r_j$$

Proof. (ii)

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$r_l = \langle P_{rl}, N_{rl}, I_{rl} \rangle$$
$$\tau \geq 0$$

To show that

$$r_j \times r_k = r_k \times r_j$$

L.H.S =

$$r_j \times r_k$$
$$\langle P_{rj}, N_{rj}, I_{rj} \rangle \times \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$= \left\langle P_{rj} \times P_{rk}, N_{rj} \times N_{rk}, \sqrt{(I_{rj})^2 + (I_{rk})^2 - (I_{rj})^2 \times (I_{rk})^2} \right\rangle$$
$$= \left\langle P_{rk} \times P_{rj}, N_{rk} \times N_{rj}, \sqrt{(I_{rk})^2 + (I_{rj})^2 - (I_{rk})^2 \times (I_{rj})^2} \right\rangle$$
$$= \langle P_{rk}, N_{rk}, I_{rk} \rangle \times \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$= r_k \times r_j$$

The proofs of 3 and 4 are straightforward as of 1 and 2.

Proof. (5)

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$
$$r_k = \langle P_{rk}, N_{rk}, I_{rk} \rangle$$
$$r_l = \langle P_{rl}, N_{rl}, I_{rl} \rangle$$
$$\tau \geq 0$$

To show that

$$\tau r_j + \tau r_k = \tau(r_j + r_k)$$

L.H.S =

$$\tau r_j + \tau r_k$$

$$\tau\langle P_{rj}, N_{rj}, I_{rj}\rangle + \tau\langle P_{rk}, N_{rk}, I_{rk}\rangle$$

$$= \left\langle \sqrt{1-\left(1-P_{rj}\right)^{\tau}}, \left(N_{rj}\right)^{\tau}, \left(I_{rj}\right)^{\tau} \right\rangle + \left\langle \sqrt{1-(1-P_{rk})^{\tau}}, (N_{rk})^{\tau}, (I_{rk})^{\tau} \right\rangle$$

$$= \left\langle \sqrt{1-\left(1-\left(P_{rj}^2+P_{rk}^2-\left(P_{rj}^2\cdot P_{rk}^2\right)\right)^{\tau}\right)}, \left(N_{rj}\cdot N_{rk}\right)^{\tau}, \left(I_{rj}\cdot I_{rk}\right)^{\tau} \right\rangle$$

R.H.S

$$\tau\left(r_j + r_k\right)$$

$$= \tau\left\langle \sqrt{\left(P_{rj}\right)^2 + (P_{rk})^2 - \left(P_{rj}\right)^2\cdot(P_{rk})^2}, N_{rj}\cdot N_{rk}, I_{rj}\cdot I_{rk} \right\rangle$$

$$= \left\langle \sqrt{1-\left(1-\left(P_{rj}^2+P_{rk}^2-\left(P_{rj}^2\cdot P_{rk}^2\right)\right)^{\tau}\right)}, \left(N_{rj}\cdot N_{rk}\right)^{\tau}, \left(I_{rj}\cdot I_{rk}\right)^{\tau} \right\rangle$$

Hence proved. ■

Proof. (6)

To prove

$$\left(\tau_j r_j + \tau_k r_k\right) = \left(\tau_j + \tau_k\right) r_j$$

$$\tau_j \geq 0$$

$$\tau_k \geq 0$$

$$\tau_j \cdot \langle P_{rj}, N_{rj}, I_{rj}\rangle + \tau_k \cdot \langle P_{rk}, N_{rk}, I_{rk}\rangle$$

$$= \left\langle \sqrt{1-\left(1-P_{rj}^2\right)^{\tau_j}}, \left(N_{rj}\right)^{\tau_j}, \left(I_{rj}\right)^{\tau_j} \right\rangle + \left\langle \sqrt{1-(1-P_{rk}^2)^{\tau_k}}, (N_{rk})^{\tau_k}, (I_{rk})^{\tau_k} \right\rangle$$

$$= \left\langle \sqrt{\left(1-\left(1-P_{rj}^2\right)^{\tau_j} + 1-(1-P_{rk}^2)^{\tau_k} - \left(1-\left(1-P_{rj}^2\right)^{\tau_j}\right)\cdot\left(1-(1-P_{rk}^2)^{\tau_k}\right)}, \left(N_{rj}\right)^{\tau_j}\cdot(N_{rk})^{\tau_k}, \left(I_{rj}\right)^{\tau_j}\cdot(I_{rk})^{\tau_k} \right\rangle$$

$$= \left\langle \sqrt{1-\left(1-P_{rj}^2\right)^{\tau_j}\cdot(1-P_{rk}^2)^{\tau_k}}, \left(N_{rj}\right)^{\tau_j+\tau_k}, \left(I_{rj}\right)^{\tau_j+\tau_k} \right\rangle$$

$$= \left\langle \sqrt{1-\left(1-P_{rj}^2\right)^{\tau_j+\tau_k}}, \left(N_{rj}\right)^{\tau_j+\tau_k}, \left(I_{rj}\right)^{\tau_j+\tau_k} \right\rangle$$

R.H.S

$$\left(\tau_j + \tau_k\right) r_j$$

$$= \left(\tau_j + \tau_k\right)\cdot\langle P_{rj}, N_{rj}, I_{rj}\rangle$$

$$= \left\langle \sqrt{1-\left(1-P_{rj}^2\right)^{\tau_j+\tau_k}}, \left(N_{rj}\right)^{\tau_j+\tau_k}, \left(I_{rj}\right)^{\tau_j+\tau_k} \right\rangle$$

Hence proved. ■

Proof. (7)

To prove

$$\left(r_j \times r_k\right)^{\tau} = r_j^{\tau} \times r_k^{\tau}$$

L.H.S

$$\left(r_j \times r_k\right)^{\tau}$$

$$\left\langle \left(P_{rj} \cdot P_{rk}\right)^{\tau}, \left(N_{rj} \cdot N_{rk}\right)^{\tau}, \sqrt{\left(I_{rj}^2 + I_{rk}^2 - I_{rj}^2 \cdot I_{rk}^2\right)^{\tau}} \right\rangle$$

$$= \left\langle \left(P_{rj} \cdot P_{rk}\right)^{\tau}, \left(N_{rj} \cdot N_{rk}\right)^{\tau}, \sqrt{1 - \left(1 - \left(I_{rj}^2 + I_{rk}^2 - I_{rj}^2 \cdot I_{rk}^2\right)\right)^{\tau}} \right\rangle$$

Now

R.H.S

$$r_j^{\tau} \times r_k^{\tau}$$

$$= \left\langle P_{rj}^{\tau}, N_{rj}^{\tau}, \sqrt{1 - \left(1 - I_{rj}^2\right)^{\tau}} \right\rangle \times \left\langle P_{rk}^{\tau}, N_{rk}^{\tau}, \sqrt{1 - (1 - I_{rk}^2)^{\tau}} \right\rangle$$

$$= \left\langle \left(P_{rj} \cdot P_{rk}\right)^{\tau}, \left(N_{rj} \cdot N_{rk}\right)^{\tau}, \sqrt{1 - \left(1 - \left(I_{rj}^2 + I_{rk}^2 - I_{rj}^2 \cdot I_{rk}^2\right)\right)^{\tau}} \right\rangle$$

Hence proved. ■

Proof. (8)

$$r_j = \langle P_{rj}, N_{rj}, I_{rj} \rangle$$

To prove that

$$\left(\left(r_j\right)^{\tau_j} + \left(r_j\right)^{\tau_k}\right) = \left(r_j\right)^{\left(\tau_j + \tau_k\right)}$$

L.H.S

$$= \langle P_{rj}, N_{rj}, I_{rj} \rangle^{\tau_j} \times \langle P_{rj}, N_{rj}, I_{rj} \rangle^{\tau_k}$$

$$\left\langle \left(P_{rj}\right)^{\tau_j}, \left(N_{rj}\right)^{\tau_j}, \sqrt{1 - \left(1 - I_{rj}^2\right)^{\tau_j}} \right\rangle \times \left\langle \left(P_{rj}\right)^{\tau_k}, \left(N_{rj}\right)^{\tau_k}, \sqrt{1 - \left(1 - I_{rj}^2\right)^{\tau_k}} \right\rangle$$

$$\left\langle \left(P_{rj}\right)^{\tau_j}\left(P_{rj}\right)^{\tau_k}, \left(N_{rj}\right)^{\tau_j}\left(N_{rj}\right)^{\tau_k}, \sqrt{\left(1 - \left(1 - I_{rj}^2\right)^{\tau_j}\right) + \left(1 - \left(1 - I_{rj}^2\right)^{\tau_k}\right) - \left(1 - \left(1 - I_{rj}^2\right)^{\tau_j}\right)\left(1 - \left(1 - I_{rj}^2\right)^{\tau_k}\right)} \right\rangle$$

$$= \left\langle \left(P_{rj}\right)^{\tau_j + \tau_k}, \left(N_{rj}\right)^{\tau_j + \tau_k}, \sqrt{1 - \left(1 - I_{rj}^2\right)^{\tau_j + \tau_k}} \right\rangle$$

R.H.S

$$= \left(r_j\right)^{\left(\tau_j + \tau_k\right)}$$

$$= \langle P_{rj}, N_{rj}, I_{rj} \rangle^{\tau_j + \tau_k}$$

$$= \left\langle \left(P_{rj}\right)^{\tau_j + \tau_k}, \left(N_{rj}\right)^{\tau_j + \tau_k}, \sqrt{1 - \left(1 - I_{rj}^2\right)^{\tau_j + \tau_k}} \right\rangle$$

Hence proved.

11 | An Innovative Technique for Solving Decision-Making Issues, relying on the Spherical Fuzzy Hypersoft Sets

In this section, we will provide a method for addressing Multiple Criteria Decision Making (MCDM) challenges built upon the spherical fuzzy hypersoft set operators. Suppose we have $\mathrm{W} = \{w_1, w_2, w_3, \dots, w_n\}$ be any limited selection of n choices, additionally, we have a limited number of m criteria, for example, $\mathrm{M} = \{m_1, m_2, m_3, \dots, m_m\}$. At the bottom of the spherical fuzzy hypersoft set, they will collect details regarding the format of $\xi = \{(\varrho, \varsigma, \varphi)\}$ the criterion in relation to the quantitative aspect of ξ is $0 \leq \varrho^2 + \varsigma^2 + \varphi^2 \leq 1$

Step 1. Data gathering: Gather the evaluative input from decision-makers organized like a matrix $G = [T_{nm}]$ as

$$\mathrm{G} = \begin{pmatrix} T_{11} & T_{12} & \cdots & T_{1m} \\ T_{21} & T_{22} & \cdots & T_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ T_{n1} & T_{n2} & \cdots & T_{nm} \end{pmatrix}$$

Step 2. Normalization: The matrix of choices $\mathrm{G} = [\mathrm{T}_{ij}]$ is converted through a normalized matrix $\overline{\mathrm{G}} = [\overline{\mathrm{T}}_{ij}]$ at this phase, employing the subsequent formulation:

$$\overline{\mathrm{T}}_{ij} = \begin{cases} \mathrm{T}_{ij}, & \text{if it concerns a benefit measure} \\ (\mathrm{T}_{ij})^c, & \text{if it concerns a cost measure} \end{cases} \tag{3}$$

where $(\mathrm{T}_{ij})^c$ denotes the complement of T_{ij}. It's crucial to bear in mind that for any SFHSS $\mathrm{T} = \{(\varrho, \varsigma), \varphi\}$, its complement is determined as

$$\mathrm{T}^c = \{(\varphi, \varsigma, \varrho)\} \tag{4}$$

Step 3. Aggregation: Aggregate the SFHSS T_{ij} (for $= 1,2,3, \dots, m$) for all alternatives W_i (for $i = 1,2,3, \dots, n$) into the preference's total worth T by using the SFHSSAA or SFHSSGA operators that have been recommended. It can be stated mathematically as:

$$\mathrm{T}_i = SFHSSAA(\mathrm{T}_{i1}, \mathrm{T}_{i2}, \mathrm{T}_{i3}, \dots, \mathrm{T}_{im}) \tag{5}$$

$$\mathrm{T}_i = SFHSSGA(\mathrm{T}_{i1}, \mathrm{T}_{i2}, \mathrm{T}_{i3}, \dots, \mathrm{T}_{im}) \tag{6}$$

Step 4. Recognize the values of the score: In line with the score's definition function, find out the score values $\mathrm{Sc}(\mathrm{T}_i)$ for all SFHSS T_i (for $i = 1,2,3, \dots, m$).

Step 5. Sorting and ordering: Arrange the alternatives W_i (for $= 1,2,3, \dots, m$) to determine the most favorable one using the score values $\mathrm{Sc}(\mathrm{T}_i)$.

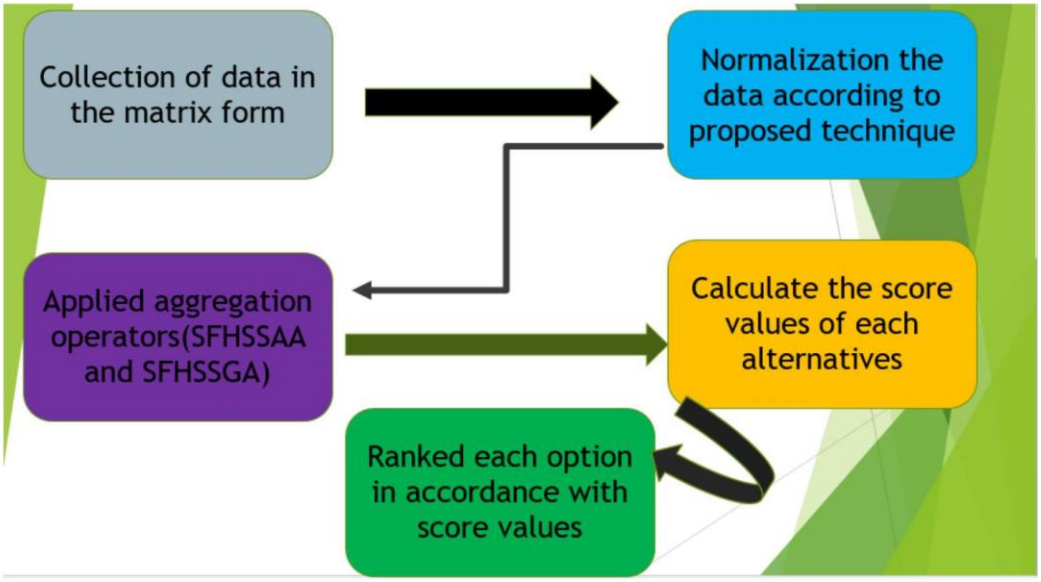

Figure 1. Create a flowchart illustrating the proposed technique.

12 | Explanatory Example

This paragraph goes into further detail about the ramifications and applicability of the mentioned technique using a laptop selection example. The reality that the established strategy could be applied to a range of lively scenarios it is not only restricted to the laptop decision problem is very vital. Innovation inside organizations produces distinct retail locations. With so many opportunities available to us, online shopping has evolved into a necessary way of life for most individuals. Without having to leave our houses, we can have what we really want. We won't have to wait in line to get anything again because installment is so convenient. Take out a bank loan for shopping money. To facilitate future transactions, a user must ascertain which laptops are popular on Tmall, Amazon, eBay, and other websites. He or she makes contact with friends who are experts in laptops. They note that the following standards are used to rank most laptops: Execution $(\mathcal{M}_1)$, shading $(\mathcal{M}_2)$, pixel $(\mathcal{M}_3)$, cost $(\mathcal{M}_4)$ and appearance $(\mathcal{M}_5)$. Then he or she selects one of the four best-rated laptop computers. However, she is undecided about which one to purchase: Apple MacBook Air $(\mathcal{N}_1)$, MacBook Pro $(\mathcal{N}_2)$, HP Spectre $(\mathcal{N}_3)$, Asus Zenbook Pro Duo $(\mathcal{N}_4)$. Clearly, the determination interplay between laptops is an MCDM problem that consists of five alternatives $\{m_1, m_2, m_3, m_4, m_5\}$, four models $\{n_1, n_2, n_3, n_4\}$ and specialist d. The developed approach can subsequently be utilized to identify the optimal layout at that moment.

Step 1. Compilation of the matrix-formatted data.

Table 1. Spherical fuzzy hypersoft set the DM that d took for $n = 2$.

	$\overline{\mathcal{M}_1}$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$
$\mathcal{N}_1$	⟨0.3,0.8,1	⟨0.8,0.5,0.3⟩	⟨0.5,0.7,0.2⟩	⟨0.6,0.2,0.3⟩	⟨0.5,0.8,0.1⟩
$\mathcal{N}_2$	⟨0.7,0.4,0.3⟩	⟨0.7,0.2,0.2⟩	⟨0.6,0.5,0.1⟩	⟨0.5,0.7,0.2⟩	⟨0.5,0.7,0.2⟩
$\mathcal{N}_3$	⟨0.2,0.9,0.2⟩	⟨0.8,0.4,0.1⟩	⟨0.2,0.9,0.2⟩	⟨0.5,0.7,0.2⟩	⟨0.8,0.3,0.3⟩
$\mathcal{N}_4$	⟨0.6,0.7,0.3⟩	⟨0.5,0.8,0.2⟩	⟨0.6,0.7,0.2⟩	⟨0.7,0.4,0.2⟩	⟨0.6,0.5,0.1⟩

Step 2. Standardize the data based on the suggested method.

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$
$\overline{\mathcal{N}_1}$	⟨0.3,0.8,0.1⟩	⟨0.8,0.5,0.3⟩	⟨0.5,0.7,0.2⟩	⟨0.3,0.2,0.6⟩	⟨0.5,0.8,0.1⟩
$\mathcal{N}_2$	⟨0.7,0.4,0.3⟩	⟨0.7,0.2,0.2⟩	⟨0.6,0.5,0.1⟩	⟨0.2,0.7,0.5⟩	⟨0.5,0.7,0.2⟩
$\mathcal{N}_3$	⟨0.2,0.9,0.2⟩	⟨0.8,0.4,0.1⟩	⟨0.2,0.9,0.	⟨0.2,0.7,	⟨0.8,0.3,0.3⟩
$\mathcal{N}_4$	⟨0.6,0.7,0.3⟩	⟨0.5,0.8,0.2⟩	⟨0.6,0.7,0.2⟩	⟨0.2,0.4,0.7⟩	⟨0.6,0.5,0.1⟩

Step 3. In this stage, we employed aggregation operators (SFHSSAA and SFHSSGA). We obtained results:

- SFHSSAA:

  $\mathcal{N}_1 = \{0.7205, 0.5737, 0.2514\}, \mathcal{N}_2 = \{0.2006, 0.7079, 0.1523\}$,

  $\mathcal{N}_3 = \{0.4126, 0.3110, 0.1543\}$, and $\mathcal{N}_4 = \{0.1823, 0.6032, 0.1532\}$

- SFHSSGA:

  $\mathcal{N}_1 = \{0.6818, 0.6820, 0.1234\}, \mathcal{N}_2 = \{0.5759, 0.7192, 0.1324\}$,

  $\mathcal{N}_3 = \{0.7633, 0.4924, 0.1565\}$, and $\mathcal{N}_4 = \{0.6444, 0.6749, 0.1013\}$

Step 4. During this phase, we computed the scores assigned to each option.

- SFHSSAA:

  $Sc(\mathcal{N}_1) = 0.1612, Sc(\mathcal{N}_2) = -0.2495, Sc(\mathcal{N}_3) = 0.0197$, and $Sc(\mathcal{N}_4) = -0.1313$.

- SFHSSGA:

  $Sc(\mathcal{N}_1) = -0.0002, Sc(\mathcal{N}_2) = -0.1576, Sc(\mathcal{N}_3) = 0.2807$, and $Sc(\mathcal{N}_4) = -0.0350$.

Step 5. Ultimately, we assigned a ranking to each option based on their respective score values.

- SFHSSAA:

$$\mathcal{N}_1 > \mathcal{N}_3 > \mathcal{N}_4 > \mathcal{N}_2$$

- SFHSSGA:

$$\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$$

As a result, the aggregate operators display the final ranks. The Apple MacBook Air is rated as having the best laptop quality out of all of its competitors by SFHSSAA. In contrast, SFHSSGA shows that, in comparison to other models, the HP Spectre achieves the best level of laptop quality. Even though both operators produce nearly identical results, their output is mediocre. However, it is imperative to emphasize that the deductions are purely conjectural in the absence of specific information regarding the laptops associated with these classifications.

13 | Conclusions and Future Work

In conclusion, creating and utilizing the Spherical Fuzzy Hypersoft Set (SFHS) has demonstrated encouraging outcomes in handling uncertainty and difficult decision-making situations. Pattern recognition, image processing, medical diagnosis, and expert systems are just a few of the domains where SFHS outperforms typical fuzzy sets by giving a strong foundation for handling vague and vague details.

Refinement of theoretical underpinnings, investigation of hybrid models using machine learning methods, and development of scalable algorithms to manage large data are the next goals for SFHS research. Moreover, information entropy and probabilistic techniques will enhance SFHS uncertainty modeling. Working with domain experts in cutting-edge fields including bioinformatics, environmental studies, Internet of Things (IoT), and social sciences will help SFHS applications take new turns. User-friendly software tools and libraries will further facilitate its uptake across disciplines.

All things considered, fuzzy logic and intelligent systems could benefit greatly from SFHS, which offers insightful information and practical answers to challenging issues in unpredictable and complicated settings.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions

Muhammad Saeed: Conceptualization of this study, Supervision, Methodology, Writing – review & editing. Mubashir Ali and Hafiz Inam Ul Haq: Investigation, Methodology, Writing - Original draft preparation. Ghluam Haider: Investigation and Writing - Original draft preparation.

Funding

This research has no funding source.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

There was no data used in the inquiry that was as stated in the article.

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