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Contra-HyperSoft Set and Contra-SuperHyperSoft Set

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Abstract

A Soft Set is a parameterized family of subsets of a universe, where each parameter selects elements relevant under that condition. A ContraSoft Set is a parameterized soft set in which each parameter's values are linked with a contradiction degree, and a threshold mechanism is applied to retain only those values that do not exceed a specified level of contradiction with respect to a chosen reference. In this paper, we explore two new concepts, namely the *Contra-HyperSoft Set* and the *Contra-SuperHyperSoft Set*, extending the framework of contradiction-aware modeling.

Keywords: Soft Set, Contradiction, ContraSoft Set, Contra-HyperSoft Set, Contra-SuperHyperSoft Set

1 | Preliminaries

We collect the basic terminology and notation used in what follows. The definitions in this paper are assumed to be finite.

1.1 | Soft Set

A Soft Set is a parameterized family of subsets selecting universe elements relevant to each parameter, supporting flexible decision modeling [1, 2, 3]. The definitions of the Soft Set are provided below.

Definition 1 (Soft Set). [1] Let U be a universal set and E a set of parameters. A *soft set* over U is defined as an ordered pair (F, E) , where F is a mapping from E to the power set $\mathcal{P}(U)$:

$$F : E \rightarrow \mathcal{P}(U).$$

For each parameter $e \in E$, $F(e) \subseteq U$ represents the set of e -approximate elements in U , with (F, E) forming a parameterized family of subsets of U .



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Example 2 (Soft Set — Hotel Filtering with Parameterized Conditions). **Universe and parameters.** Let the universe of candidate hotels be

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}.$$

Let the parameter set be

$$E = \{\text{near_station}, \text{free_breakfast}, \text{onsen}, \text{under}\yen12000, \text{twin_room}\}.$$

Define a soft set (F, E) with $F : E \rightarrow \mathcal{P}(U)$ by

$$\begin{aligned} F(\text{near_station}) &= \{h_1, h_2, h_5, h_7\}, \\ F(\text{free_breakfast}) &= \{h_2, h_4, h_5, h_6\}, \\ F(\text{onsen}) &= \{h_3, h_5, h_8\}, \\ F(\text{under}\yen12000) &= \{h_1, h_4, h_5, h_7, h_8\}, \\ F(\text{twin_room}) &= \{h_2, h_4, h_6, h_7\}. \end{aligned}$$

Concrete queries and explicit computations.

$$\begin{aligned} \text{(i) near_station \& under}\yen12000: & F(\text{near_station}) \cap F(\text{under}\yen12000) \\ &= \{h_1, h_2, h_5, h_7\} \cap \{h_1, h_4, h_5, h_7, h_8\} \\ &= \{h_1, h_5, h_7\}, \quad |\cdot| = 3. \end{aligned}$$

$$\begin{aligned} \text{(ii) (free_breakfast} \cup \text{onsen) \& twin_room:} & (F(\text{free_breakfast}) \cup F(\text{onsen})) \cap F(\text{twin_room}) \\ &= (\{h_2, h_4, h_5, h_6\} \cup \{h_3, h_5, h_8\}) \cap \{h_2, h_4, h_6, h_7\} \\ &= \{h_2, h_4, h_6\}, \quad |\cdot| = 3. \end{aligned}$$

$$\text{(iii) onsen \& twin_room: } F(\text{onsen}) \cap F(\text{twin_room}) = \{h_3, h_5, h_8\} \cap \{h_2, h_4, h_6, h_7\} = \emptyset.$$

These results illustrate how a soft set supports multi-criterion filtering by standard set operations with exact outputs.

1.2 | ContraSoft Set

A ContraSoft Set is a parameterized soft set where each parameter's values are associated with a contradiction degree, and thresholding is used to aggregate only those values that are not too contradictory with respect to a chosen reference[4]. This allows soft-set modeling to filter or weight information based on contradiction, rather than uncertainty.

Definition 3 (Contradiction on attribute values). [4] Let V be a nonempty finite set of attribute values. A *contradiction function* on V is a map

$$c : V \times V \longrightarrow [0, 1]$$

such that

$$c(v, v) = 0 \quad (\text{reflexivity}), \quad c(v, w) = c(w, v) \quad (\text{symmetry}).$$

The quantity $c(v, w)$ measures the degree of *contradiction* between v and w (larger means more contradictory).

Example 4 (Contradiction on attribute values — temperature preference). Let $V = \{\text{cold}, \text{mild}, \text{hot}\}$. Define the symmetric contradiction $c : V \times V \rightarrow [0, 1]$ (with $c(v, v) = 0$) by

	cold	mild	hot
cold	0	0.4	0.9
mild	0.4	0	0.5
hot	0.9	0.5	0

so, e.g., $c(\text{cold}, \text{hot}) = 0.9$ expresses a strong contradiction, while $c(\text{cold}, \text{mild}) = 0.4$ is moderate.

Definition 5 (ContraSoft structure). [4] Let U be a nonempty universe and E a nonempty set of parameters. For each $e \in E$ fix:

- a nonempty finite value set V_e ;

- a contradiction function $c_e : V_e \times V_e \rightarrow [0, 1]$ (Definition 3);
- a designated *reference value* $v_e^* \in V_e$.

Write $V := \bigsqcup_{e \in E} (\{e\} \times V_e)$ for the disjoint union of all parameter–value pairs.

Example 6 (ContraSoft structure — hotels by noise and price). Let the universe be $U = \{h_1, h_2, h_3, h_4\}$ and parameters $E = \{\text{noise}, \text{price}\}$. For each $e \in E$ fix a finite value-set V_e , a contradiction $c_e : V_e \times V_e \rightarrow [0, 1]$, and a reference value $v_e^* \in V_e$:

$V_{\text{noise}} = \{\text{quiet}, \text{moderate}, \text{loud}\},$	$c_{\text{noise}} =$		quiet	moderate	loud	, $v_{\text{noise}}^* = \text{quiet}.$
		quiet	0	0.3	0.8	
		moderate	0.3	0	0.4	
		loud	0.8	0.4	0	
$V_{\text{price}} = \{\text{cheap}, \text{mid}, \text{expensive}\},$	$c_{\text{price}} =$		cheap	mid	expensive	, $v_{\text{price}}^* = \text{mid}.$
		cheap	0	0.2	0.7	
		mid	0.2	0	0.3	
		expensive	0.7	0.3	0	

These choices realize Definition (ContraSoft structure) by specifying value domains, their contradiction degrees, and per-parameter references.

Definition 7 (ContraSoft Set). [4] Let U be a finite universe of objects and E a finite set of parameters. A *ContraSoft Set* is a quadruple

$$\text{CS} := (U, E, F, c),$$

where

- $F : E \rightarrow \mathcal{P}(U)$ is the (crisp) soft mapping; $F(e) \subseteq U$ is the set of objects *accepted* (or classified as positive) under parameter e ;
- $c : E \times E \rightarrow [0, 1]$ is a *contradiction degree* on parameters, symmetric and reflexive on the diagonal:

$$c(e, e) = 0, \quad c(e, f) = c(f, e) \quad (\forall e, f \in E).$$

For $x \in U$ and $e \in E$, the atomic lemma “ x is accepted by e ” is represented by

$$A(x, e) : x \in F(e),$$

with truth value **T** if $x \in F(e)$ and **F** otherwise.

Example 8 (ContraSoft Set — Noise-Aware Hotel Selection with Contradiction Thresholding). **Universe, parameters, and soft mapping.** Let the same universe U be as above. Consider parameters

$$E = \{\text{quiet}, \text{nightlife}, \text{coworking}, \text{scenic}\}.$$

Define $F : E \rightarrow \mathcal{P}(U)$ by

$$\begin{aligned} F(\text{quiet}) &= \{h_1, h_3, h_5, h_8\}, & F(\text{nightlife}) &= \{h_2, h_4, h_6\}, \\ F(\text{coworking}) &= \{h_2, h_5, h_6, h_7\}, & F(\text{scenic}) &= \{h_3, h_5, h_7, h_8\}. \end{aligned}$$

Contradiction degrees on parameters. Let $c : E \times E \rightarrow [0, 1]$ be symmetric with

$c(\cdot, \cdot)$	quiet	nightlife	coworking	scenic
quiet	0	0.9	0.4	0.2
nightlife	0.9	0	0.3	0.5
coworking	0.4	0.3	0	0.4
scenic	0.2	0.5	0.4	0

(diagonal 0, larger values mean more contradictory).

Reference and thresholded aggregation. Fix the *reference parameter* $e^* = \text{quiet}$ and threshold $\tau = 0.4$. Define the *accepted envelope*

$$S^{(\tau)}(e^*) := \bigcup_{e \in E: c(e, e^*) \leq \tau} F(e).$$

Eligible parameters are those within the contradiction radius:

$$c(\text{quiet}, \text{quiet}) = 0, \quad c(\text{coworking}, \text{quiet}) = 0.4, \quad c(\text{scenic}, \text{quiet}) = 0.2, \quad c(\text{nightlife}, \text{quiet}) = 0.9 > \tau.$$

Hence

$$\begin{aligned} S^{(\tau)}(\text{quiet}) &= F(\text{quiet}) \cup F(\text{coworking}) \cup F(\text{scenic}) \\ &= \{h_1, h_3, h_5, h_8\} \cup \{h_2, h_5, h_6, h_7\} \cup \{h_3, h_5, h_7, h_8\} \\ &= \{h_1, h_2, h_3, h_5, h_6, h_7, h_8\}, \quad |S^{(\tau)}| = 7. \end{aligned}$$

Tighter threshold for comparison. With $\tau' = 0.2$, only quiet and scenic are admitted:

$$S^{(\tau')}(\text{quiet}) = F(\text{quiet}) \cup F(\text{scenic}) = \{h_1, h_3, h_5, h_7, h_8\}, \quad |S^{(\tau')}| = 5.$$

Thus $S^{(\tau)}$ is monotone in τ , and the contradiction metric controls how widely we aggregate across potentially conflicting parameters.

1.3 | HyperSoft Set and SuperHyperSoft Set

HyperSoft Set maps each multi-attribute tuple from a Cartesian product to a subset of the universe consistent with those values [5, 6, 7, 8, 9]. Related concepts such as the Fuzzy HyperSoft Set [10, 11] and the HyperSoft Expert Set [12] are also known. SuperHyperSoft Set maps tuples of subsets from power-set domains to universe subsets, generalizing HyperSoft; singletons in each coordinate recover HyperSoft [13, 14, 15, 16].

Definition 9 (HyperSoft Set). [5] Let U be a finite universe and let $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m$ be m attribute value domains. Consider the Cartesian product

$$\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_m,$$

so that each parameter $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_m) \in \mathcal{C}$ chooses a single value $\gamma_i \in \mathcal{A}_i$ for every attribute. A *HyperSoft Set* over U is a pair $(G, \mathcal{C})$ where

$$G : \mathcal{C} \longrightarrow \mathcal{P}(U)$$

assigns to each multi-attribute parameter γ a subset $G(\gamma) \subseteq U$. Equivalently,

$$(G, \mathcal{C}) = \{(\gamma, G(\gamma)) : \gamma \in \mathcal{C}\}.$$

Example 10 (HyperSoft Set — Multi-Attribute Restaurant Finder). **Universe and attributes.** Let the universe of candidate restaurants be

$$U = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9\}.$$

Let the attribute domains be

$$\mathcal{A}_1 = \{\text{jpn}, \text{ita}, \text{ind}\}, \quad \mathcal{A}_2 = \{\text{low}, \text{mid}, \text{high}\}, \quad \mathcal{A}_3 = \{\text{omn}, \text{veg}, \text{vgn}\}.$$

The parameter space is the Cartesian product $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$. A HyperSoft Set is a mapping $G : \mathcal{C} \rightarrow \mathcal{P}(U)$ that assigns a subset of restaurants to each single-valued tuple (cuisine, price, diet).

Specification (nonempty images).

$$\begin{aligned} G(\text{jpn}, \text{mid}, \text{veg}) &= \{r_2, r_5\}, & G(\text{jpn}, \text{low}, \text{omn}) &= \{r_1, r_3\}, \\ G(\text{ita}, \text{mid}, \text{veg}) &= \{r_4\}, & G(\text{ind}, \text{low}, \text{vgn}) &= \{r_6, r_8\}, \\ G(\text{ita}, \text{high}, \text{omn}) &= \{r_7, r_9\}, \end{aligned}$$

and $G(\gamma) = \emptyset$ for all other $\gamma \in \mathcal{C}$.

Concrete queries with exact set calculations.

(i) Exactly (jpn, mid, veg) : $G(\text{jpn}, \text{mid}, \text{veg}) = \{r_2, r_5\}, |\cdot| = 2.$

(ii) Union of two precise asks: $G(\text{jpn}, \text{low}, \text{omn}) \cup G(\text{ita}, \text{high}, \text{omn})$
 $= \{r_1, r_3\} \cup \{r_7, r_9\} = \{r_1, r_3, r_7, r_9\}, |\cdot| = 4.$

(iii) Disjointness of incompatible tuples: $G(\text{jpn}, \text{low}, \text{omn}) \cap G(\text{ind}, \text{low}, \text{vgn}) = \{r_1, r_3\} \cap \{r_6, r_8\} = \emptyset.$

The HyperSoft Set captures *single-value* choices per attribute; each tuple pinpoints a crisp slice of U .

Definition 11 (SuperHyperSoft Set). [13, 16] Let U be a finite universe. Let $a_1, a_2, \dots, a_n$ be distinct attributes with finite, pairwise disjoint value-sets $A_1, A_2, \dots, A_n$ (i.e., $A_i \cap A_j = \emptyset$ for $i \neq j$). Write $\mathcal{P}(A_i)$ for the power set of A_i and form

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n).$$

A *SuperHyperSoft Set* over U is a pair $(F, \mathcal{C})$ with

$$F : \mathcal{C} \longrightarrow \mathcal{P}(U),$$

so that for each $\gamma = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathcal{C}$ (where $\alpha_i \subseteq A_i$) we have a subset $F(\gamma) \subseteq U$. Formally,

$$(F, \mathcal{C}) = \{(\gamma, F(\gamma)) : \gamma \in \mathcal{C}, F(\gamma) \subseteq U\}.$$

Example 12 (SuperHyperSoft Set — Flexible Restaurant Finder with Set-Valued Coordinates). **Universe and attributes.** Use the same U and attribute value-sets $A_1 = \{\text{jpn}, \text{ita}, \text{ind}\}$, $A_2 = \{\text{low}, \text{mid}, \text{high}\}$, $A_3 = \{\text{omn}, \text{veg}, \text{vgn}\}$. In the SuperHyperSoft setting, the parameter space is

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3),$$

so each coordinate is a *subset* of admissible values (a flexible filter).

Mapping (nonempty images). Define $F : \mathcal{C} \rightarrow \mathcal{P}(U)$ by

$$\begin{aligned} F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\}) &= \{r_2, r_4, r_5\}, \\ F(\{\text{ind}\}, \{\text{low}, \text{mid}\}, \{\text{vgn}\}) &= \{r_6, r_8\}, \\ F(\{\text{jpn}, \text{ind}\}, \{\text{low}\}, \{\text{omn}, \text{veg}\}) &= \{r_1, r_3, r_6\}, \\ F(\{\text{ita}\}, \{\text{high}\}, \{\text{omn}\}) &= \{r_7, r_9\}, \end{aligned}$$

and $F(\alpha) = \emptyset$ otherwise.

Reading the parameters. For example, $\alpha = (\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\})$ means: cuisine is Japanese *or* Italian, price is low *or* mid, diet is vegetarian. Then $F(\alpha) = \{r_2, r_4, r_5\}$ is the recommended subset.

Coherence with HyperSoft via singletons. If we restrict to singletons in each coordinate, SuperHyperSoft reduces to HyperSoft. Concretely,

$$F(\{\text{jpn}\}, \{\text{mid}\}, \{\text{veg}\}) = \{r_2, r_5\} = G(\text{jpn}, \text{mid}, \text{veg}),$$

so the singleton tuple reproduces the HyperSoft slice exactly. Moreover,

$$F(\{\text{jpn}\}, \{\text{mid}\}, \{\text{veg}\}) \subseteq F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\}) = \{r_2, r_4, r_5\},$$

exhibiting the intended *flexible expansion* when coordinates are broadened from single values to sets of values.

Cardinality checks.

$$|F(\{\text{jpn}, \text{ita}\}, \{\text{low}, \text{mid}\}, \{\text{veg}\})| = 3, \quad |F(\{\text{ind}\}, \{\text{low}, \text{mid}\}, \{\text{vgn}\})| = 2.$$

Thus SuperHyperSoft enables compact specification of multi-value preferences per attribute and directly returns the filtered subset of U .

2 | Main Results

In this section, we present and analyze the principal outcomes of our study.

2.1 | Contra-HyperSoft Set

Contra-HyperSoft Set augments HyperSoft with a tuple-wise contradiction metric, reference selector, and threshold, uniting parameter slices within the admissible radius.

Definition 13 (Coordinatewise contradiction). Let $\mathcal{A}_1, \dots, \mathcal{A}_m$ be nonempty finite sets. For each $i \in \{1, \dots, m\}$ a *contradiction function* is a map

$$c_i : \mathcal{A}_i \times \mathcal{A}_i \longrightarrow [0, 1] \quad \text{with} \quad c_i(a, a) = 0, \quad c_i(a, b) = c_i(b, a).$$

When needed for exact reductions, we assume the *zero-separation* property $c_i(a, b) = 0 \Rightarrow a = b$.

Definition 14 (Tuple-level contradiction). Let $\mathcal{C} := \mathcal{A}_1 \times \dots \times \mathcal{A}_m$ and write $\gamma = (\gamma_1, \dots, \gamma_m)$, $\delta = (\delta_1, \dots, \delta_m) \in \mathcal{C}$. Define the aggregated contradiction by

$$\Delta(\gamma, \delta) := \max_{1 \leq i \leq m} c_i(\gamma_i, \delta_i) \in [0, 1].$$

Then $\Delta(\gamma, \delta) = \Delta(\delta, \gamma)$ and $\Delta(\gamma, \gamma) = 0$. If each c_i is zero-separating, then $\Delta(\gamma, \delta) = 0 \iff \gamma = \delta$.

Definition 15 (Reference selector). A *reference selector* is a map $\rho : \mathcal{C} \rightarrow \mathcal{C}$. Two canonical choices are

$$(\text{self-centered}) \quad \rho(\gamma) = \gamma, \quad (\text{fixed-reference}) \quad \rho(\gamma) \equiv r \quad \text{for a fixed } r \in \mathcal{C}.$$

Definition 16 (Contra-HyperSoft Set (CHS)). Let U be a finite universe and let $G : \mathcal{C} \rightarrow \mathcal{P}(U)$ be a HyperSoft mapping. Fix contradiction kernels $\{c_i\}_{i=1}^m$, a reference selector ρ , and a threshold $\tau \in [0, 1]$. The associated *Contra-HyperSoft Set* is the tuple

$$\text{CHS} := (U, \{\mathcal{A}_i, c_i\}_{i=1}^m, G, \rho, \tau),$$

together with the *filtered mapping*

$$G_\rho^{(\tau)} : \mathcal{C} \longrightarrow \mathcal{P}(U), \quad G_\rho^{(\tau)}(\gamma) := \bigcup_{\delta \in \mathcal{C} : \Delta(\delta, \rho(\gamma)) \leq \tau} G(\delta).$$

Example 17 (Contra-HyperSoft Set — Candidate Shortlisting under Conflicting Signals (self-centered selector)). **Universe and attributes.** Let the candidate pool be $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$. Consider three single-valued attribute domains:

$$\mathcal{A}_1 = \{\text{junior, mid, senior}\}, \quad \mathcal{A}_2 = \{\text{onsite, hybrid, remote}\}, \quad \mathcal{A}_3 = \{\text{backend, frontend, data}\}.$$

The parameter space is $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$.

Coordinatewise contradictions. All c_i are symmetric with 0 on the diagonal.

$$\begin{aligned} c_1 &= \begin{array}{c|ccc} & \text{junior} & \text{mid} & \text{senior} \\ \hline \text{junior} & 0 & 0.4 & 0.9 \\ \text{mid} & 0.4 & 0 & 0.4 \\ \text{senior} & 0.9 & 0.4 & 0 \end{array}, \\ c_2 &= \begin{array}{c|ccc} & \text{onsite} & \text{hybrid} & \text{remote} \\ \hline \text{onsite} & 0 & 0.3 & 0.8 \\ \text{hybrid} & 0.3 & 0 & 0.3 \\ \text{remote} & 0.8 & 0.3 & 0 \end{array}, \\ c_3 &= \begin{array}{c|ccc} & \text{backend} & \text{frontend} & \text{data} \\ \hline \text{backend} & 0 & 0.5 & 0.4 \\ \text{frontend} & 0.5 & 0 & 0.6 \\ \text{data} & 0.4 & 0.6 & 0 \end{array}. \end{aligned}$$

Aggregate tuple-contradiction: $\Delta(\gamma, \delta) := \max\{c_1(\gamma_1, \delta_1), c_2(\gamma_2, \delta_2), c_3(\gamma_3, \delta_3)\}$.

HyperSoft mapping $G : \mathcal{C} \rightarrow \mathcal{P}(U)$ (**nonempty images**).

$$\begin{aligned} G(\text{junior, remote, frontend}) &= \{c_1, c_3\}, & G(\text{mid, hybrid, backend}) &= \{c_2, c_5\}, \\ G(\text{senior, onsite, data}) &= \{c_4\}, & G(\text{mid, remote, data}) &= \{c_6\}, \\ G(\text{senior, hybrid, backend}) &= \{c_7, c_8\}. \end{aligned}$$

CHS filter. Choose the *self-centered* selector $\rho(\gamma) = \gamma$ and threshold $\tau = 0.4$. Let $\gamma^* = (\text{mid}, \text{hybrid}, \text{backend})$. Compute $\Delta(\cdot, \gamma^*)$ on the above tuples:

δ	$\Delta(\delta, \gamma^*)$
(mid, hybrid, backend)	$\max(0, 0, 0) = 0 \ (\leq \tau)$
(senior, hybrid, backend)	$\max(0.4, 0, 0) = 0.4 \ (\leq \tau)$
(mid, remote, data)	$\max(0, 0.3, 0.4) = 0.4 \ (\leq \tau)$
(senior, onsite, data)	$\max(0.4, 0.3, 0.4) = 0.4 \ (\leq \tau)$
(junior, remote, frontend)	$\max(0.4, 0.3, 0.5) = 0.5 \ (> \tau)$

Hence

$$G_{\rho}^{(\tau)}(\gamma^*) = \{c_2, c_5\} \cup \{c_7, c_8\} \cup \{c_6\} \cup \{c_4\} = \{c_2, c_4, c_5, c_6, c_7, c_8\}, \quad |\cdot| = 6.$$

With a tighter threshold $\tau' = 0.3$, only the base slice survives: $G_{\rho}^{(\tau')}(\gamma^*) = \{c_2, c_5\}$, illustrating monotonicity in τ .

Example 18 (Contra-HyperSoft Set — Travel Package Selection (fixed reference)). **Universe and attributes.** Let $U = \{\text{pkg}_1, \dots, \text{pkg}_{10}\}$ be travel packages. Attributes:

$$\mathcal{A}_1 = \{\text{winter, spring, summer, autumn}\}, \quad \mathcal{A}_2 = \{\text{ski, beach, culture}\}, \quad \mathcal{A}_3 = \{\text{solo, couple, family}\}.$$

Contradiction matrices (symmetric, 0 on diagonal).

		winter	spring	summer	autumn	
$c_1 =$	winter	0	0.3	0.8	0.5	
	spring	0.3	0	0.3	0.5	
	summer	0.8	0.3	0	0.3	
	autumn	0.5	0.5	0.3	0	
		ski	beach	culture		
$c_2 =$	ski	0	0.9	0.4	,	
	beach	0.9	0	0.4		
	culture	0.4	0.4	0		
		solo	couple	family		
$c_3 =$	solo	0	0.2	0.6	.	
	couple	0.2	0	0.3		
	family	0.6	0.3	0		

Aggregate $\Delta(\gamma, \delta) := \max\{c_1(\gamma_1, \delta_1), c_2(\gamma_2, \delta_2), c_3(\gamma_3, \delta_3)\}$.

HyperSoft mapping (nonempty images).

$$\begin{aligned} G(\text{winter, ski, family}) &= \{\text{pkg}_1, \text{pkg}_2\}, & G(\text{summer, beach, couple}) &= \{\text{pkg}_3, \text{pkg}_4\}, \\ G(\text{spring, culture, solo}) &= \{\text{pkg}_5\}, & G(\text{autumn, culture, family}) &= \{\text{pkg}_6, \text{pkg}_7\}, \\ G(\text{summer, culture, family}) &= \{\text{pkg}_8\}, & G(\text{winter, beach, solo}) &= \{\text{pkg}_9\}. \end{aligned}$$

CHS filter (fixed reference). Choose the fixed reference $r = (\text{summer, beach, family})$ and threshold $\tau = 0.5$. Evaluate $\Delta(\cdot, r)$:

γ	$\Delta(\gamma, r)$
(winter, ski, family)	$\max(0.8, 0.9, 0) = 0.9 \ (> \tau)$
(summer, beach, couple)	$\max(0, 0, 0.3) = 0.3 \ (\leq \tau)$
(spring, culture, solo)	$\max(0.3, 0.4, 0.6) = 0.6 \ (> \tau)$
(autumn, culture, family)	$\max(0.3, 0.4, 0) = 0.4 \ (\leq \tau)$
(summer, culture, family)	$\max(0, 0.4, 0) = 0.4 \ (\leq \tau)$
(winter, beach, solo)	$\max(0.8, 0, 0.6) = 0.8 \ (> \tau)$

Thus the accepted tuples are the 2nd, 4th, and 5th. The CHS envelope at r is

$$G_{\rho=r}^{(\tau)}(r) = \{\text{pkg}_3, \text{pkg}_4\} \cup \{\text{pkg}_6, \text{pkg}_7\} \cup \{\text{pkg}_8\} = \{\text{pkg}_3, \text{pkg}_4, \text{pkg}_6, \text{pkg}_7, \text{pkg}_8\},$$

with cardinality 5. If we tighten to $\tau' = 0.3$, only the 2nd tuple remains, so $G_{\rho \equiv r}^{(\tau')}(r) = \{\text{pkg}_3, \text{pkg}_4\}$, demonstrating the control afforded by the contradiction threshold.

Proposition 19 (Basic properties). *For fixed $(U, \{\mathcal{A}_i, c_i\}, G, \rho)$ the family $\{G_\rho^{(\tau)}\}_{\tau \in [0,1]}$ is monotone in τ : if $0 \leq \tau_1 \leq \tau_2 \leq 1$ then $G_\rho^{(\tau_1)}(\gamma) \subseteq G_\rho^{(\tau_2)}(\gamma)$ for all $\gamma \in \mathcal{C}$. Moreover $G_\rho^{(1)}(\gamma) = \bigcup_{\delta \in \mathcal{C}} G(\delta)$ for all γ .*

Proof: If $\tau_1 \leq \tau_2$ then $\{\delta : \Delta(\delta, \rho(\gamma)) \leq \tau_1\} \subseteq \{\delta : \Delta(\delta, \rho(\gamma)) \leq \tau_2\}$, hence the unions are nested. For $\tau = 1$ the constraint is vacuous since $\Delta \in [0, 1]$. $\square$

Theorem 20 (CHS generalizes the HyperSoft Set). *Assume each c_i is zero-separating and take the self-centered selector $\rho(\gamma) = \gamma$. Then for $\tau = 0$ one has*

$$G_\rho^{(0)}(\gamma) = G(\gamma) \quad (\forall \gamma \in \mathcal{C}).$$

Proof: By definition, $G_\rho^{(0)}(\gamma) = \bigcup_{\delta: \Delta(\delta, \gamma) \leq 0} G(\delta)$. Since $\Delta(\delta, \gamma) \geq 0$ always, the inequality forces $\Delta(\delta, \gamma) = 0$. Zero-separation gives $\delta = \gamma$, thus the union is $G(\gamma)$. $\square$

Definition 21 (Neighborhood-based ContraSoft on a single attribute). Let V be a finite set with contradiction $c : V \times V \rightarrow [0, 1]$, and let $F : V \rightarrow \mathcal{P}(U)$. For $\tau \in [0, 1]$, the *neighborhood-based* ContraSoft transform is

$$F^{(\tau)}(v) := \bigcup_{w \in V: c(w, v) \leq \tau} F(w) \quad (v \in V).$$

Fixing $v^* \in V$ yields the *fixed-reference* variant $F^{(\tau; v^*)}(v) := \bigcup_{w: c(w, v^*) \leq \tau} F(w)$.

Example 22 (Neighborhood-based ContraSoft — Destination Selection by Climate Preference). **Universe and attribute.** Let the universe of candidate destinations be

$$U = \{u_1 = \text{Reykjavik}, u_2 = \text{Zurich}, u_3 = \text{Lisbon}, u_4 = \text{Dubai}, u_5 = \text{Helsinki}, u_6 = \text{Vancouver}\}.$$

Consider a single attribute “preferred climate” with value set

$$V = \{\text{cold}, \text{mild}, \text{warm}, \text{hot}\}.$$

Contradiction on V . Let $c : V \times V \rightarrow [0, 1]$ be symmetric with $c(v, v) = 0$:

	cold	mild	warm	hot
cold	0	0.3	0.6	0.9
mild	0.3	0	0.3	0.7
warm	0.6	0.3	0	0.3
hot	0.9	0.7	0.3	0

Baseline soft mapping. Define $F : V \rightarrow \mathcal{P}(U)$ by

$$F(\text{cold}) = \{u_1, u_5\}, \quad F(\text{mild}) = \{u_2, u_6\}, \quad F(\text{warm}) = \{u_3\}, \quad F(\text{hot}) = \{u_4\}.$$

Neighborhood-based ContraSoft transform. For threshold $\tau \in [0, 1]$ and center $v \in V$,

$$F^{(\tau)}(v) = \bigcup_{w \in V: c(w, v) \leq \tau} F(w).$$

Case 1 (moderate neighborhood). Let $v = \text{mild}$ and $\tau = 0.35$. Eligible neighbors satisfy $c(w, \text{mild}) \leq 0.35$:

$$c(\text{cold}, \text{mild}) = 0.3 \ (\checkmark), \quad c(\text{mild}, \text{mild}) = 0 \ (\checkmark), \quad c(\text{warm}, \text{mild}) = 0.3 \ (\checkmark), \quad c(\text{hot}, \text{mild}) = 0.7 \ (\times).$$

Therefore

$$\begin{aligned} F^{(0.35)}(\text{mild}) &= F(\text{cold}) \cup F(\text{mild}) \cup F(\text{warm}) \\ &= \{u_1, u_5\} \cup \{u_2, u_6\} \cup \{u_3\} = \{u_1, u_2, u_3, u_5, u_6\}, \quad |\cdot| = 5. \end{aligned}$$

Case 2 (tight neighborhood). Let $\tau = 0.20$. Only $w = \text{mild}$ satisfies $c(w, \text{mild}) \leq 0.20$, hence

$$F^{(0.20)}(\text{mild}) = F(\text{mild}) = \{u_2, u_6\}, \quad |\cdot| = 2.$$

These computations show how increasing τ expands the accepted neighborhood in V and unions the corresponding destination sets in U .

Theorem 23 (CHS generalizes ContraSoft). *Suppose $m = 1$, so $\mathcal{C} = \mathcal{A}_1 =: V$, and let $c_1 = c$. Identify $G : V \rightarrow \mathcal{P}(U)$ with F . Then:*

(a) *With the self-centered selector $\rho(v) = v$, one has*

$$G_\rho^{(\tau)}(v) = \bigcup_{w: c(w,v) \leq \tau} G(w) = F^{(\tau)}(v), \quad \forall v \in V.$$

(b) *With the fixed-reference selector $\rho(v) \equiv v^*$, one has*

$$G_\rho^{(\tau)}(v) = \bigcup_{w: c(w,v^*) \leq \tau} G(w) = F^{(\tau;v^*)}(v), \quad \forall v \in V.$$

Hence, for $m = 1$ the CHS construction recovers both standard ContraSoft variants.

Proof: When $m = 1$, $\Delta(w, \rho(v)) = c(w, \rho(v))$. Substituting $\rho(v) = v$ gives (a); substituting $\rho(v) \equiv v^*$ gives (b). The set-theoretic unions agree by definition in both cases. $\square$

2.2 | Contra-SuperHyperSoft Set

Contra-SuperHyperSoft Set extends to set-valued coordinates, using lifted subset contradictions and aggregate radius; selector-threshold filtering unions nearby SuperHyperSoft slices effectively.

Definition 24 (Base and lifted contradictions). Let $A_1, \dots, A_m$ be nonempty finite sets of attribute values and let $c_i : A_i \times A_i \rightarrow [0, 1]$ be *contradiction functions* (symmetric and reflexive: $c_i(a, a) = 0$, $c_i(a, b) = c_i(b, a)$). Assume the *zero-separation* property $c_i(a, b) = 0 \Rightarrow a = b$. For subsets $S, T \subseteq A_i$ define the *lifted* contradiction

$$\hat{c}_i(S, T) := \begin{cases} \max \left\{ \max_{a \in S} \min_{b \in T} c_i(a, b), \max_{b \in T} \min_{a \in S} c_i(a, b) \right\}, & S, T \neq \emptyset, \\ 0, & S = T = \emptyset, \\ 1, & \text{otherwise.} \end{cases}$$

(With finiteness, the max/min are attained.)

Lemma 25 (Symmetry, reflexivity, and zero-separation on subsets). *For each i and $S, T \subseteq A_i$:*

- (a) $\hat{c}_i(S, T) = \hat{c}_i(T, S)$ and $\hat{c}_i(S, S) = 0$.
- (b) *If c_i is zero-separating, then $\hat{c}_i(S, T) = 0$ implies $S = T$.*

Proof: (a) Symmetry follows by exchanging the two max terms; reflexivity is immediate. (b) If $\hat{c}_i(S, T) = 0$ with $S, T \neq \emptyset$, then $\max_{a \in S} \min_{b \in T} c_i(a, b) = 0$ and $\max_{b \in T} \min_{a \in S} c_i(a, b) = 0$. Thus, for each $a \in S$ there is $b \in T$ with $c_i(a, b) = 0$, hence $a = b$ by zero-separation, so $S \subseteq T$. The second equality gives $T \subseteq S$. The cases with empties are by definition. $\square$

Definition 26 (Product parameter space and aggregate contradiction). Let $\mathcal{C} := \mathcal{P}(A_1) \times \dots \times \mathcal{P}(A_m)$ and write $\alpha = (\alpha_1, \dots, \alpha_m)$, $\beta = (\beta_1, \dots, \beta_m) \in \mathcal{C}$. Define the tuple-level contradiction by

$$\Delta(\alpha, \beta) := \max_{1 \leq i \leq m} \hat{c}_i(\alpha_i, \beta_i) \in [0, 1].$$

Then $\Delta(\alpha, \beta) = \Delta(\beta, \alpha)$ and $\Delta(\alpha, \alpha) = 0$; if each c_i is zero-separating, then by Lemma 25 we have $\Delta(\alpha, \beta) = 0 \iff \alpha = \beta$.

Definition 27 (Reference selector). A *reference selector* is any map $\rho : \mathcal{C} \rightarrow \mathcal{C}$. Two common choices are the *self-centered* selector $\rho(\alpha) = \alpha$ and a *fixed-reference* selector $\rho(\alpha) \equiv r$ for a fixed $r \in \mathcal{C}$.

Definition 28 (Contra-SuperHyperSoft Set (CSHS)). Let U be a finite universe and let $F : \mathcal{C} \rightarrow \mathcal{P}(U)$ be a SuperHyperSoft mapping. Fix contradiction kernels $\{c_i\}$, their lifts $\{\hat{c}_i\}$, an aggregate Δ , a selector ρ , and a threshold $\tau \in [0, 1]$. The associated *Contra-SuperHyperSoft Set* is the tuple

$$\text{CSHS} := (U, \{A_i, c_i\}_{i=1}^m, F, \rho, \tau),$$

together with the *filtered* mapping

$$F_\rho^{(\tau)} : \mathcal{C} \longrightarrow \mathcal{P}(U), \quad F_\rho^{(\tau)}(\alpha) := \bigcup_{\beta \in \mathcal{C} : \Delta(\beta, \rho(\alpha)) \leq \tau} F(\beta).$$

Example 29 (CSHS in E-commerce Fraud Review (self-centered selector)). **Setup.** Let the universe of orders be $U = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8\}$. Take two attribute domains:

$$A_1 = \{\text{card}, \text{crypto}\}, \quad A_2 = \{\text{verified}, \text{partial}, \text{missing}\}.$$

Base contradictions $c_1, c_2 : [\cdot] \rightarrow [0, 1]$ (symmetric, 0 on the diagonal):

$$c_1 = \begin{array}{c|cc} & \text{card} & \text{crypto} \\ \hline \text{card} & 0 & 0.8 \\ \text{crypto} & 0.8 & 0 \end{array}, \quad c_2 = \begin{array}{c|ccc} & \text{verified} & \text{partial} & \text{missing} \\ \hline \text{verified} & 0 & 0.3 & 0.9 \\ \text{partial} & 0.3 & 0 & 0.6 \\ \text{missing} & 0.9 & 0.6 & 0 \end{array}.$$

Lift $\hat{c}_i$ to subsets by Definition (lifted contradiction) and aggregate

$$\Delta((S_1, S_2), (T_1, T_2)) := \max\{\hat{c}_1(S_1, T_1), \hat{c}_2(S_2, T_2)\}.$$

The SuperHyperSoft mapping $F : \mathcal{P}(A_1) \times \mathcal{P}(A_2) \rightarrow \mathcal{P}(U)$ is specified by

$$\begin{aligned} F(\{\text{crypto}\}, \{\text{missing}\}) &= \{o_1, o_2\}, & F(\{\text{crypto}\}, \{\text{partial}\}) &= \{o_3\}, \\ F(\{\text{card}\}, \{\text{missing}\}) &= \{o_4\}, & F(\{\text{card}\}, \{\text{partial}\}) &= \{o_5, o_6\}, \\ F(\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\}) &= \{o_7\}, & \text{all other pairs} &\text{map to } \emptyset. \end{aligned}$$

Choose the *self-centered* selector $\rho(\alpha) = \alpha$ and threshold $\tau = 0.6$.

Filtering at $\alpha_0 = (\{\text{crypto}\}, \{\text{missing}\})$. To avoid overfull lines, we list the computations in an aligned display:

$$\begin{aligned} \Delta((\{\text{crypto}\}, \{\text{missing}\}), \alpha_0) &= \max(0, 0) = 0 \leq \tau, \\ \Delta((\{\text{crypto}\}, \{\text{partial}\}), \alpha_0) &= \max(0, c_2(\text{partial}, \text{missing})) = \max(0, 0.6) = 0.6 \leq \tau, \\ \Delta((\{\text{card}\}, \{\text{missing}\}), \alpha_0) &= \max(0.8, 0) = 0.8 > \tau, \\ \Delta((\{\text{card}\}, \{\text{partial}\}), \alpha_0) &= \max(0.8, 0.6) = 0.8 > \tau, \end{aligned}$$

$$\begin{aligned} \Delta((\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\}), \alpha_0) &= \max(\hat{c}_1(\{\text{card}, \text{crypto}\}, \{\text{crypto}\}), \hat{c}_2(\{\text{partial}, \text{missing}\}, \{\text{missing}\})) \\ &= \max(0.8, 0.6) = 0.8 > \tau. \end{aligned}$$

Hence

$$F_\rho^{(\tau)}(\alpha_0) = \{o_1, o_2\} \cup \{o_3\} = \{o_1, o_2, o_3\}, \quad |F_\rho^{(\tau)}(\alpha_0)| = 3.$$

Filtering at $\alpha_1 = (\{\text{card}\}, \{\text{partial}\})$.

β	$\Delta(\beta, \alpha_1)$
$(\{\text{card}\}, \{\text{partial}\})$	$0 \leq \tau$
$(\{\text{card}\}, \{\text{missing}\})$	$\max(0, 0.6) = 0.6 \leq \tau$
$(\{\text{crypto}\}, \{\text{partial}\})$	$\max(0.8, 0) = 0.8 > \tau$
$(\{\text{crypto}\}, \{\text{missing}\})$	$\max(0.8, 0.6) = 0.8 > \tau$
$(\{\text{card}, \text{crypto}\}, \{\text{partial}, \text{missing}\})$	$\max(0.8, 0.6) = 0.8 > \tau$

Therefore

$$F_\rho^{(\tau)}(\alpha_1) = \{o_5, o_6\} \cup \{o_4\} = \{o_4, o_5, o_6\}, \quad |F_\rho^{(\tau)}(\alpha_1)| = 3.$$

This illustrates how the CSHS envelope aggregates nearby subset-parameters under the contradiction metric.

Example 30 (CSHS in Cloud Deployment Recommendation (self-centered selector)). **Setup.** Let the universe of candidate nodes be $U = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7\}$. Attributes:

$$A_1 = \{\text{us-east}, \text{us-west}, \text{eu}\}, \quad A_2 = \{\text{cpu}, \text{gpu}, \text{memory}\}.$$

Base contradictions (0 on the diagonal, symmetric):

$$c_1 = \begin{array}{c|ccc} & \text{us-east} & \text{us-west} & \text{eu} \\ \hline \text{us-east} & 0 & 0.4 & 0.7 \\ \text{us-west} & 0.4 & 0 & 0.8 \\ \text{eu} & 0.7 & 0.8 & 0 \end{array}, \quad c_2 = \begin{array}{c|ccc} & \text{cpu} & \text{gpu} & \text{memory} \\ \hline \text{cpu} & 0 & 0.5 & 0.3 \\ \text{gpu} & 0.5 & 0 & 0.7 \\ \text{memory} & 0.3 & 0.7 & 0 \end{array}.$$

Lift to subsets by $\hat{c}_i$ and aggregate by $\Delta((S_1, S_2), (T_1, T_2)) = \max\{\hat{c}_1(S_1, T_1), \hat{c}_2(S_2, T_2)\}$.

SuperHyperSoft mapping F (nonempty images shown):

$$\begin{aligned} F(\{\text{us-east}\}, \{\text{cpu}\}) &= \{n_1, n_2\}, & F(\{\text{us-east}\}, \{\text{gpu}\}) &= \{n_3\}, \\ F(\{\text{us-west}\}, \{\text{cpu}\}) &= \{n_4\}, & F(\{\text{eu}\}, \{\text{cpu}\}) &= \{n_5\}, \\ F(\{\text{us-east}, \text{us-west}\}, \{\text{cpu}\}) &= \{n_6\}, \\ F(\{\text{us-east}\}, \{\text{cpu}, \text{gpu}\}) &= \{n_7\}. \end{aligned}$$

Choose the *self-centered* selector $\rho(\alpha) = \alpha$ and threshold $\tau = 0.5$.

Filtering at $\alpha^* = (\{\text{us-east}\}, \{\text{cpu}\})$. For each β with $F(\beta) \neq \emptyset$, compute $\Delta(\beta, \alpha^*)$:

β	$\Delta(\beta, \alpha^*)$
$(\{\text{us-east}\}, \{\text{cpu}\})$	$\max(0, 0) = 0 \leq \tau$
$(\{\text{us-east}\}, \{\text{gpu}\})$	$\max(0, c_2(\text{gpu}, \text{cpu}) = 0.5) = 0.5 \leq \tau$
$(\{\text{us-west}\}, \{\text{cpu}\})$	$\max(c_1(\text{us-west}, \text{us-east}) = 0.4, 0) = 0.4 \leq \tau$
$(\{\text{eu}\}, \{\text{cpu}\})$	$\max(0.7, 0) = 0.7 > \tau$
$(\{\text{us-east}, \text{us-west}\}, \{\text{cpu}\})$	$\max(\hat{c}_1(\{\text{us-east}, \text{us-west}\}, \{\text{us-east}\}) = 0.4, 0) = 0.4 \leq \tau$
$(\{\text{us-east}\}, \{\text{cpu}, \text{gpu}\})$	$\max(0, \hat{c}_2(\{\text{cpu}, \text{gpu}\}, \{\text{cpu}\}) = 0.5) = 0.5 \leq \tau$

Hence

$$F_\rho^{(\tau)}(\alpha^*) = \{n_1, n_2\} \cup \{n_3\} \cup \{n_4\} \cup \{n_6\} \cup \{n_7\} = \{n_1, n_2, n_3, n_4, n_6, n_7\},$$

with $|F_\rho^{(\tau)}(\alpha^*)| = 6$. Nodes requiring a European region are excluded by the contradiction bound.

Proposition 31 (Monotonicity in the threshold). *If $0 \leq \tau_1 \leq \tau_2 \leq 1$, then $F_\rho^{(\tau_1)}(\alpha) \subseteq F_\rho^{(\tau_2)}(\alpha)$ for all $\alpha \in \mathcal{C}$. Moreover, $F_\rho^{(1)}(\alpha) = \bigcup_{\beta \in \mathcal{C}} F(\beta)$ for all α .*

Proof: The index sets $\{\beta : \Delta(\beta, \rho(\alpha)) \leq \tau\}$ are nested as τ grows, hence so are the unions. For $\tau = 1$ the constraint is vacuous since $\Delta \in [0, 1]$. $\square$

Theorem 32 (CSHS generalizes SuperHyperSoft). *Assume zero-separation for each c_i and take the self-centered selector $\rho(\alpha) = \alpha$. Then for $\tau = 0$ we have*

$$F_\rho^{(0)}(\alpha) = F(\alpha) \quad (\forall \alpha \in \mathcal{C}).$$

Proof: By definition, $F_\rho^{(0)}(\alpha) = \bigcup_{\beta: \Delta(\beta, \alpha) \leq 0} F(\beta)$. Since $\Delta \geq 0$, the inequality forces $\Delta(\beta, \alpha) = 0$. By Definition 26 and Lemma 25, this occurs iff $\beta = \alpha$. Hence the union collapses to $F(\alpha)$. $\square$

Definition 33 (Singleton embedding of HyperSoft into SuperHyperSoft). Let $\iota : A_1 \times \cdots \times A_m \rightarrow \mathcal{C}$ be

$$\iota(\gamma_1, \dots, \gamma_m) := (\{\gamma_1\}, \dots, \{\gamma_m\}).$$

We say $F : \mathcal{C} \rightarrow \mathcal{P}(U)$ is *singleton-supported* for $G : A_1 \times \cdots \times A_m \rightarrow \mathcal{P}(U)$ if $F(\iota(\gamma)) = G(\gamma)$ for all γ and $F(\alpha) = \emptyset$ whenever some α_i is not a singleton.

Lemma 34 (Compatibility of contradictions on singletons). *For any $a, b \in A_i$ we have $\hat{c}_i(\{a\}, \{b\}) = c_i(a, b)$. Consequently, for $\gamma, \delta \in A_1 \times \dots \times A_m$,*

$$\Delta(\iota(\gamma), \iota(\delta)) = \max_{1 \leq i \leq m} c_i(\gamma_i, \delta_i).$$

Proof: From Definition 24 with singletons, $\max\{\min c_i(a, b), \min c_i(b, a)\} = c_i(a, b)$ by symmetry, and the product case follows. $\square$

Theorem 35 (CSHS generalizes Contra-HyperSoft). *Let $G : A_1 \times \dots \times A_m \rightarrow \mathcal{P}(U)$ be a HyperSoft set and F be a singleton-supported extension (Definition 33). Let ρ_H be a selector on $A_1 \times \dots \times A_m$ and define ρ_S on $\mathcal{C}$ by $\rho_S(\iota(\gamma)) := \iota(\rho_H(\gamma))$, with arbitrary values elsewhere. Then for every γ and $\tau \in [0, 1]$,*

$$F_{\rho_S}^{(\tau)}(\iota(\gamma)) = \bigcup_{\delta: \max_i c_i(\delta_i, \rho_H(\gamma)_i) \leq \tau} G(\delta) =: G_{\rho_H}^{(\tau)}(\gamma),$$

i.e. the CSHS filtered mapping restricted to singleton parameters coincides with the Contra-HyperSoft mapping for G built from $\{c_i\}$ and the same threshold.

Proof: By Definition 28,

$$F_{\rho_S}^{(\tau)}(\iota(\gamma)) = \bigcup_{\beta: \Delta(\beta, \rho_S(\iota(\gamma))) \leq \tau} F(\beta).$$

Since F is singleton-supported, only β of the form $\iota(\delta)$ contribute. By Lemma 34 and the definition of ρ_S , $\Delta(\iota(\delta), \rho_S(\iota(\gamma))) = \max_i c_i(\delta_i, \rho_H(\gamma)_i)$. Substitute and use $F(\iota(\delta)) = G(\delta)$ to obtain the stated equality. $\square$

Theorem 36 (CSHS generalizes ContraSoft). *Let $m = 1$ with base set $A_1 =: V$ and contradiction $c := c_1$. Let $F : \mathcal{P}(V) \rightarrow \mathcal{P}(U)$ be singleton-supported for some $F_0 : V \rightarrow \mathcal{P}(U)$ via $F(\{v\}) = F_0(v)$ and $F(S) = \emptyset$ if $|S| \neq 1$. Then, with the self-centered selector, for all $v \in V$ and $\tau \in [0, 1]$,*

$$F_{\text{id}}^{(\tau)}(\{v\}) = \bigcup_{w: c(w, v) \leq \tau} F_0(w),$$

which is exactly the neighborhood-based ContraSoft transform on (V, c) . With a fixed reference $v^* \in V$, the same construction yields the fixed-reference ContraSoft variant.

Proof: Specialize Theorem 35 to $m = 1$ with $G \equiv F_0$ and observe that $\Delta(\{w\}, \{v\}) = \hat{c}_1(\{w\}, \{v\}) = c(w, v)$. Singleton support reduces the union to singletons, yielding the stated form. $\square$

3 | Conclusion

In this paper, we examined the concepts of the ContraSoft Set, the Contra-HyperSoft Set, and the Contra-SuperHyperSoft Set.

Future work will focus on extending these ideas by integrating them with richer frameworks, including Neutrosophic Sets[17], Plithogenic Sets[18], Uncertain Sets[19, 20], Rough Sets[21], and TreeSoft Sets[22]. Moreover, we anticipate the development of generalized structures that apply these contra-based approaches to Graphs[23], HyperGraphs[24], SuperHyperGraphs[25, 26, 27], and broader HyperStructures[28, 29, 30]. Such investigations are expected to open new directions in handling contradiction-aware representations across diverse domains.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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