

Paper Type: Original Article

Uncertainty Modeling and Evidence Combination: DST and DS_mT in Perspective

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Received: 23 Aug 2025

Revised: 03 Nov 2025

Accepted: 09 Feb 2026

Published: 10 Feb 2026

Abstract

Dempster-Shafer Theory (DST) and Dezert-Smarandache Theory (DS_mT) are prominent frameworks within evidence theory for managing uncertainty and fusing information. While both utilize basic belief assignments (BBAs) on sets of hypotheses, they diverge critically in their foundational mathematical assumptions and conflict handling. This article provides a comprehensive comparison, detailing the philosophical and practical shift from DST to its generalized superset, DS_mT. DST is built upon the power set 2^Θ of a mutually exclusive frame of discernment, forcing conflict to be resolved via global normalization (Dempster's Rule), which can lead to counter-intuitive results in high-conflict scenarios (e.g., Zadeh's paradox). DS_mT, conversely, operates on the hyper-power set D^Θ of an exhaustive but non-exclusive frame, allowing for the representation of vague, paradoxical, or overlapping concepts (e.g., $A \cap B$). Crucially, DS_mT employs the Proportional Conflict Redistribution (PCR) family of rules, which resolves disagreement by locally redistributing conflicting mass back only to the propositions that generated it. This mechanism ensures stability and interpretability, even under extreme conflict. We analyze the trade-offs in expressiveness and computational cost, illustrating that the choice between the two theories hinges on two core factors: the expected level of conflict and the degree of conceptual overlap in the problem domain. Ultimately, DS_mT is demonstrated to fully encompass DST, functioning as a robust, flexible alternative for modern multi-sensor fusion, intelligence analysis, and decision-making systems characterized by ambiguity and high disagreement.

Keywords: Belief Functions, Evidence Theory, Dempster-Shafer Theory (DST), Dezert-Smarandache Theory (DS_mT), Information Fusion, Conflict Management, Basic Belief Assignment (BBA), Proportional Conflict Redistribution (PCR), Power Set, Hyper-Power Set.

1 | Introduction

Both Dempster-Shafer Theory (DST) and Dezert-Smarandache Theory (DS_mT) belong to the broader family of evidence-theoretic approaches that sit between classic probability theory and fuzzy logic. They share the notion of allocating “mass” to sets of hypotheses rather than to singletons, but they diverge sharply in the mathematical structure they assume for those sets and in how they treat contradictions.

1.1 | Where the two theories come from

Dempster-Shafer Theory (DST) traces its roots to Arthur P. Dempster's 1967 paper “Upper and Lower Probabilities” [1] and was later formalized by Glenn Shafer in his 1976 monograph “A Mathematical Theory



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<https://doi.org/10.61356/j.hsse.2026.5644>



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of Evidence” [10]. These works introduced the belief-function formalism that allocates mass to subsets of a mutually exclusive frame of discernment and combined evidence with Dempster’s rule.

Dezert-Smarandache Theory (DSMT) was presented by Jean Dezert and Florentin Smarandache in articles and conferences, e.g. [2-8, 11-14]. The theory was subsequently expanded in a series of collected papers, “Advances and Applications of DSMT for Information Fusion” [15-19] where the authors relaxed the exclusivity assumption, introduced the hyper-power set [4], and proposed the Proportional Conflict Redistribution (PCR) family of combination rules.

1.2 | Philosophical and Mathematical Motivations

1. *From probability to belief* – Classical probability forces a single numeric value on each elementary event. DST introduced the idea that an analyst may reserve part of the belief for a set of possibilities when the evidence is insufficient to pinpoint a singleton. This “ignorance mass” is represented by the assignment to the whole frame Θ [7, 9].
2. *From exclusivity to openness* – Early applications of DST (e.g., early sensor fusion for radar/sonar) often assumed that the elementary hypotheses were mutually exclusive (e.g., “target is a plane” vs. “target is a ship”). Real-world problems soon revealed situations where concepts overlap (e.g., “vehicle may be civilian and possibly hostile”) or where paradoxes arise (two experts giving contradictory categorical statements). DSMT was proposed precisely to remove the exclusivity requirement and to allow the intersection of hypotheses to carry belief.
3. *From global normalization to local conflict handling* – Dempster’s rule resolves conflict by normalizing away the total conflicting mass (the factor $1 - K$). When K is large, the resulting redistribution can be unintuitive (Zadeh’s paradox) [21, 22]. DSMT’s PCR family instead identifies each partial conflict and redistributes it only to the propositions that generated it, preserving the semantic meaning of the disagreement.

1.3 | Representative Application Domains

The Table 1 below provides a comparative overview of typical DST vs. DSMT applications across diverse domains.

Table 1. Comparative overview of typical DST vs. DSMT applications across diverse domains.

Domain	Typical DST usage (strengths)	Typical DSMT usage (strengths)
Multisensor target tracking (radar, sonar, EO)	Low-to-moderate conflict scenarios where targets can be cleanly classified (e.g., “aircraft” vs. “missile”).	Situations with ambiguous signatures (e.g., stealth objects that appear simultaneously as “friendly” and “unknown”) or when sensors report contradictory categories.
Intelligence analysis & threat assessment	Structured analytic techniques where analysts agree on a fixed taxonomy of threats.	Open-ended threat vocabularies, overlapping geopolitical labels (“state-actor”, “non-state actor”, “proxy”), and high-conflict testimony from disparate sources.
Medical diagnosis	Cases where symptoms map to mutually exclusive disease codes (ICD-10).	Complex syndromes where a patient may satisfy criteria for several overlapping conditions (e.g., autoimmune disorders with shared biomarkers).
Robotics & autonomous navigation	Fusion of lidar and camera detections when obstacles are clearly “occupied” vs. “free”.	Environments with semantic ambiguity (e.g., “glass surface that could be a wall or a window”) requiring belief on intersecting spatial concepts.
Information security & intrusion detection	Correlating alerts that are clearly “malicious” vs. “benign”.	Dealing with adversarial evasion where logs contain contradictory indicators (e.g., a process flagged both as trusted and suspicious).
Social media sentiment & opinion mining	Simple polarity classification (positive/negative/neutral).	Nuanced opinions where a post can be simultaneously sarcastic and supportive, demanding belief on intersecting sentiment tags.

Decision support for disaster management	Aggregating sensor readings (e.g., flood level vs. dry) with low conflict.	Combining heterogeneous reports (citizen reports, satellite imagery, weather models) that often contradict each other, especially in the early phases of a crisis.
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Across these domains, the choice between DST and DSMT hinges on two practical questions:

1. Are the elementary hypotheses truly exclusive? If the answer is “yes,” DST’s simpler power-set machinery is usually sufficient and computationally cheaper.
2. Is the evidence expected to be highly conflicting or vague? If conflict is expected to be strong, DSMT’s PCR rules provide more stable and interpretable fusion results.

1.4 | Core Differences Between DST and DSMT

- *Mathematical substrate* – DST works on the power set (union-only) of an exclusive frame; DSMT works on the hyper-power set (union + intersection) of an exhaustive but non-exclusive frame.
- *Conflict resolution* – DST uses global normalization (Dempster’s rule); DSMT uses local proportional redistribution (PCR5, PCR6, ...).
- *Expressiveness* – DSMT can encode paradoxical or overlapping propositions (e.g., “Friend $\wedge$ Foe”) that DST forces to zero.
- *Computational cost* – The hyper-power set grows faster than the ordinary power set, so DSMT can be more demanding; however, many practical implementations prune irrelevant intersections, keeping the workload manageable.

Understanding these foundations equips researchers and engineers to select the right belief-function framework for their specific uncertainty-handling challenge, and to appreciate why DSMT is often described as a generalized superset of DST.

2 | Frames of Discernment: From Exclusive to Overlapping Hypotheses

Before diving into the detailed comparison, it helps to see side-by-side how the two frameworks differ in their underlying assumptions, set structures, conflict-resolution mechanisms, and typical use cases.

The Table 2 below summarizes these key distinctions.

Table 2. Fundamental contrasts between Dempster-Shafer Theory and Dezert-Smarandache Theory.

Feature	Dempster-Shafer Theory (DST)	Dezert-Smarandache Theory (DSMT)
Model / Frame of Discernment	Shafer’s model – The frame of discernment is a set called Θ that contains the elementary hypotheses $\theta_1, \theta_2, \dots, \theta_n$. In this model the hypotheses must be mutually exclusive (no two can be true at the same time) and exhaustive (together they cover all possibilities).	Free DSMT model (or Hybrid DSMT model) – The frame of discernment is again a set Θ with elements $\theta_1, \theta_2, \dots, \theta_n$, but the only requirement is that the hypotheses are exhaustive. They do not have to be mutually exclusive; they may overlap. The hybrid version allows you to declare particular pairs as exclusive when you know that they really cannot coexist.
Set algebra (what collections of hypotheses can be used)	Power set – denoted 2^Θ . This is the collection of all subsets of Θ . Because the hypotheses are exclusive, any subset is just a union of distinct elementary hypotheses.	Hyper-power set – denoted D^Θ . This structure contains all possible unions and intersections of the elementary hypotheses, including expressions such as $\theta_i \cap \theta_j$ (the overlap of two hypotheses). Thus you can assign belief to intersecting or paradoxical concepts that are forbidden in the ordinary power set.

Conflict handling	Dempster's rule of combination – First the conjunctive (joint) mass is computed. Then the total conflict mass K (the sum of all products that lead to the empty set) is removed and the remaining masses are normalized by dividing by $1 - K$. When K is large, the normalization inflates the surviving masses and can give unintuitive results (e.g., Zadeh's paradox).	Proportional Conflict Redistribution (PCR) rules – e.g., PCR5 or PCR6. Each partial conflict (a product that would have gone to the empty set) is identified, and that conflict mass is redistributed only to the two propositions that generated it, in proportion to the original masses they contributed. No global normalization is performed, so the empty-set mass stays zero and the result remains stable even under extreme conflict.
Typical applicability	Works well when the problem has low conflict and the hypotheses can be cleanly separated (mutually exclusive).	Designed for high-conflict, imprecise, or vague situations where hypotheses may overlap or where the exclusivity assumption does not hold.

2.1 | Why the distinction matters

In many real-world sensing or classification problems, concepts are fuzzy. A radar track might be “possibly a civilian aircraft” and “possibly a hostile drone” simultaneously, reflecting overlapping interpretations. DST forbids assigning belief to such intersections because the empty set is assumed to represent total contradiction. DSMT, by contrast, treats these intersections as legitimate focal elements, preserving the nuance of the evidence.

3 | Combination Rules and Conflict Management

In this study, Fourth-Derivative Hybrid Block Methods (FDHBMs) were proposed, analyzed, and applied to the numerical solution of first-order Initial Value Problems in Ordinary Differential Equations. The one-step and two-step FDHBM schemes derived through interpolation and collocation attained order nineteen with favorable error constants. The methods were shown to be consistent and zero-stable, satisfying the fundamental requirements of numerical integration techniques.

3.1 | Dempster's Rule (DST) [20]

1. Conjunctive step – multiply masses of compatible focal elements (those whose intersection is non-empty).
2. Normalization – compute the total conflict mass

$$K = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$$

and redistribute it uniformly across all non-empty propositions:

$$m_{\text{DST}}(C) = \frac{1}{1 - K} \sum_{A \cap B = C} m_1(A) m_2(B)$$

3. Effect of high conflict – when K approaches 1, the denominator $1 - K$ becomes tiny, inflating the remaining masses and often yielding counter-intuitive results (the classic Zadeh paradox).

3.2 | Proportional Conflict Redistribution (PCR) – DSMT's answer to conflict

DSMT introduces a family of PCR rules (PCR5, PCR6, ...). The most widely used, PCR5, proceeds as follows for two sources:

- Conjunctive mass – same as in Dempster's rule, but without immediate normalization.

- Partial conflict identification – each pair (A, B) for which the intersection of A and B is empty generates a partial conflict mass equal to the product of the two basic belief assignments, $m_1(A) \times m_2(B)$.
- Redistribution – the partial conflict is split only between the two focal elements that generated it.

Let the **Frame of Discernment** be $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}, n \geq 2$, of exhaustive elements.

Let $m_1 : \Theta \rightarrow [0,1]$ and $m_2 : \Theta \rightarrow [0,1]$ be two **masses**.

The, the **Conjunctive Rule** of these two masses is defined as:

$$m_{12}(X) = (m_1 \oplus m_2)(X) = \sum_{\substack{Y, Z \in G \\ Y \cap Z = X}} m_1(Y) m_2(Z)$$

The **Hyper-Power Set** G^Θ is defined in DSMT as:

1. $\phi, \theta_1, \theta_2, \dots, \theta_n \in G^\Theta$.
2. If $A, B \in G^\Theta$, then $A \cap B \in G^\Theta$ and $A \cup B \in G^\Theta$.
3. If $A \in G^\Theta$, then the complement of $A, G^\Theta - A \in G^\Theta$.
4. No other elements belong to G^Θ , except those obtained by using rules 1, 2, or 3.
5. Then the PCR5 fusion rule of proportionally redistributing the conflicting masses to their original contributions is given by the general formula:

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \phi}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right]$$

Whence, by replacing X with A and respectively with B, we get the values for $m_{PCR5}(A)$ and $m_{PCR5}(B)$ respectively.

The vacuous BBA does not have a neutral impact in the fusion process when applying PCR5 and PCR6, which is often considered counter-intuitive (unlike Dempster's rule), and this is why the PCR6+ rule was developed, as it satisfies this expected and desirable property. {Empty set mass – remains zero; no global normalization is performed}

3.3 | Key Consequences

- Localized redistribution preserves the intuition that conflict should reinforce the propositions actually involved.
- Stability under extreme conflict – even when K is close to 1, PCR5 yields sensible results because the conflict is not forced into unrelated hypotheses.
- Compatibility – when the frame is truly exclusive (Shafer's model) and conflict is low, PCR5 reduces to the conjunctive rule, making DST a special case of DSMT.

4 | Comparative Applicability

To help decide which belief-fusion framework best fits a given problem, the following matrix matches typical situation characteristics with the theory that generally delivers more reliable and efficient results [20]. The Table 3 below provides a comparative applicability of Dempster-Shafer Theory vs. Dezert-Smarandache Theory.

Table 3. Comparative applicability of Dempster-Shafer Theory vs. Dezert-Smarandache Theory.

Situation	Prefer DST	Prefer DS _m T
Low-conflict environments (e.g., well-calibrated sensors with clear categories)	✓ ☐ Simpler computation, familiar semantics	–
High conflict (e.g., contradictory expert opinions, adversarial data)	✗ ☐ Normalization can distort results	✓ ☐ PCR5/6 keep conflict meaningful
Vague or overlapping concepts (e.g., “partially friendly”, “uncertain terrain”)	✗ ☐ Cannot assign belief to intersections	✓ ☐ Hyper-power set captures overlaps
Need for explicit exclusivity constraints (e.g., legal categories that must not intersect)	✓ ☐ Built-in exclusivity	✓ ☐ Hybrid DS _m model can enforce selected exclusivities while keeping flexibility elsewhere
Real-time processing with limited resources	✓ ☐ Fewer combinatorial terms (power set)	– (hyper-power set grows faster, but efficient approximations exist)

5 | Conceptual Summary – How DST and DS_mT Relate, Differ, and Complement One Another

In this study, Fourth-Derivative Hybrid Block Methods (FDHBMs) were proposed, analyzed, and applied to the numerical solution of first-order Initial Value Problems in Ordinary Differential Equations. The one-step and two-step FDHBM schemes derived through interpolation and collocation attained order nineteen with favorable error constants. The methods were shown to be consistent and zero-stable, satisfying the fundamental requirements of numerical integration techniques.

5.1 | Model Hierarchy – DST as a Special Case of DS_mT

Shafer’s exclusive model (the backbone of DST) assumes that every elementary hypothesis in the frame of discernment is both exhaustive and mutually exclusive. In other words, exactly one of the hypotheses must be true and no two can be true together.

The hybrid DS_m model starts from a free model where only exhaustiveness is required; mutual exclusivity is optional. When you deliberately declare every pair of hypotheses to be exclusive inside the hybrid model, you obtain exactly the Shafer model.

Any algorithm, proof, or software component that works for the hybrid DS_m model can be run on a purely exclusive frame without modification; the extra expressive power simply collapses to the familiar DST setting. Conversely, a DST-only implementation cannot handle frames where exclusivity is violated, because its underlying set algebra (the ordinary power set) lacks the necessary intersection elements.

5.1.1 | Practical Tip

If you are building a generic fusion engine, implement the hyper-power set and the PCR combination rules once. When you later encounter a problem with a strictly exclusive taxonomy, you can switch on a “exclusivity flag” and let the engine behave exactly like DST, saving you from maintaining two separate code bases.

5.2 | Conflict Philosophy – From Global Error to Valuable Information

The following table (Table 4) illustrates the shift from global normalization to local redistribution within these hybrid systems.

Table 4. Conflict Handling in DST versus DS_mT: From Global Normalisation to Local Proportional Redistribution.

Aspect	Dempster-Shafer Theory (DST)	Dezert-Smarandache Theory (DS _m T)
What conflict means	A global inconsistency that must be eliminated before a coherent belief can be reported.	A signal that tells you which propositions are at odds; the conflict itself carries semantic weight.
How it is handled	Compute the total conflict mass K (the sum of all products that lead to the empty set). Then normalize the remaining masses by dividing by $1 - K$. The empty-set mass disappears completely.	Identify each partial conflict (e.g., the product of a mass on A with a mass on B where $A \cap B = \emptyset$). Redistribute that partial mass only to A and B , in proportion to the original contributions (PCR5, PCR6, ...). The empty-set mass stays zero; no global scaling occurs.
Resulting behaviour	When K is large, the normalization step can dramatically inflate the belief in propositions that were not directly involved in the conflict, sometimes producing paradoxical conclusions (the classic Zadeh example).	Even under extreme disagreement, the belief remains anchored to the propositions that generated the conflict, preserving a more intuitive interpretation of the data.

5.2.1 | Why the Philosophical Shift Matters

1. *Interpretability* – Analysts can trace back the final belief to the exact sources of disagreement, which is crucial in safety-critical domains (e.g., aviation, medical diagnosis).
2. *Robustness* – PCR rules remain numerically stable when conflict approaches 100%; DST's denominator ($1 - K$) can become vanishingly small, leading to overflow or meaningless probabilities.
3. *Decision-making* – When conflict is treated as information, a decision maker can explicitly weigh “how much the sources disagree” as a factor in risk assessment, rather than seeing it disappear after normalization.

5.3 | Mathematical Structures – Power Set vs. Hyper-Power Set

Power set (2Θ) – Contains only unions of the elementary hypotheses. Because the hypotheses are exclusive, any subset corresponds to a straightforward “either/or” statement. The size of the power set grows as 2^n , where n is the number of hypotheses.

Hyper-power set ($D\Theta$) – Enriches the algebra by adding all possible intersections in addition to unions. This yields a lattice that includes expressions such as “ $\theta_1 \cap \theta_2$ ”, “ $\theta_1 \cap \theta_2 \cap \theta_3$ ”, and any combination thereof. The cardinality grows super-exponentially, but many practical problems involve only a small subset of these intersections, allowing efficient pruning.

5.3.1 | Expressive Gains

1. *Paradoxical statements* – You can assign belief to a proposition that is simultaneously true for two otherwise contradictory hypotheses (e.g., “the target is both Friend and Foe”). In DST this intersection would be forced to the empty set, erasing the nuance.
2. *Vagueness and overlap* – Concepts like “partially friendly” or “uncertain terrain” can be modeled as intersections of base hypotheses, preserving the degree of overlap directly in the belief representation.
3. *Hybrid modelling* – The hybrid DS_m model lets you declare specific exclusivity constraints (e.g., “Friend and Foe are exclusive, but Friend and Neutral may overlap”). The hyper-power set accommodates both the exclusive and overlapping parts in a single unified structure.

5.3.2 | Computational Considerations

- Although the hyper-power set is larger, modern DSMT implementations use focused belief propagation: they generate only those intersections that actually receive non-zero mass from the input BBAs. This dramatically reduces the effective state space.
- For very large frames, approximation schemes (e.g., pruning low-mass focal elements, clustering similar hypotheses) keep the computation tractable while retaining the essential expressive benefits.

5.4 | Putting It All Together – When to View DSMT as a Superset

Because the hybrid DSMT model can be configured to enforce any pattern of exclusivity, and because its PCR combination rules reduce to Dempster’s rule when conflict is negligible, DSMT truly subsumes DST:

- If your problem satisfies the strict exclusivity and low-conflict assumptions, you can safely operate in the DST mode of DSMT, gaining the simplicity of the power set and the familiar Dempster combination.
- If you encounter any of the following, you should switch to the full DSMT machinery:
 - High conflict ($K > 0.5$) between sources.
 - Overlapping or vague concepts that cannot be expressed as disjoint sets.
 - Requirements to preserve the provenance of conflict for audit or explainability.

In practice, many real-world systems start with a DST-style configuration and upgrade to DSMT only when the data reveal its limitations. This incremental approach lets developers leverage existing DST tools while retaining a clear migration path to the richer DSMT framework.

5.5 | Take-away Checklist

- *Model choice* – Ask yourself: Are my hypotheses mutually exclusive? If yes, DST may suffice; if not, adopt the hybrid DSMT model.
- *Conflict level* – Estimate the expected conflict mass K . If K is likely to be high, plan for PCR5/PCR6 rather than Dempster’s rule.
- *Expressiveness needs* – Do you need to represent intersections (e.g., “Friend $\wedge$ Neutral”) or paradoxical statements? If yes, you need the hyper-power set.
- *Computational budget* – For small frames, the hyper-power set is manageable; for large frames, use pruning or hybrid exclusivity to limit growth.

By keeping these points in mind, you can navigate smoothly between DST and DSMT, exploiting the strengths of each while avoiding their pitfalls.

6 | Practical Illustration

To further ground these theoretical concepts in a real-world context, it is helpful to examine a practical application of data fusion. In multisensor environments, individual sensors often provide conflicting or indeterminate evidence regarding the identity of a target. The following table (Table 5) presents the initial Basic Belief Assignments (BBAs) from two distinct sensors, illustrating the distribution of certainty, neutrality, and global ignorance within a hybrid system.

Table 5. Sensors and their basic belief assignments.

Sensor	Assigned belief (basic belief assignment)
Sensor A	$m_A(\text{Friend}) = 0.6$ $m_A(\text{Friend} \wedge \text{Neutral}) = 0.3$ $m_A(\Theta) = 0.1$
Sensor B	$m_B(\text{Foe}) = 0.7$ $m_B(\text{Neutral}) = 0.2$ $m_B(\Theta) = 0.1$

6.1 | What happens under Dempster-Shafer Theory (DST)

1. Intersection rule – In DST the elementary hypotheses are required to be mutually exclusive. Therefore, the intersection $\text{Friend} \wedge \text{Foe}$ is defined as the empty set (i.e., it cannot occur).
2. Conflict mass (K) – All products of belief masses that involve incompatible hypotheses contribute to the conflict. In this example the dominant conflicting term is the product of Sensor A's 0.6 mass on Friend with Sensor B's 0.7 mass on Foe:
3. $K \approx 0.6 \times 0.7 = 0.42$
(Other smaller conflicting products add a little more, but the total K is roughly 0.42.)
4. Normalization – Dempster's rule removes the conflict mass K and rescales the remaining masses by dividing by $(1 - K)$. Because K is large, the denominator $(1 - K)$ is small (≈ 0.58), which inflates the surviving beliefs. The result can end up giving a relatively high belief to the Neutral proposition, even though the two sensors are strongly disagreeing about whether the target is a Friend or a Foe. This is the classic “counter-intuitive” outcome associated with high conflict in DST.

6.2 | What happens under Dezert-Smarandache Theory (DSmT) – free model

1. *No exclusivity constraint* – DSmT does not force Friend and Foe to be mutually exclusive. Their intersection $\text{Friend} \wedge \text{Foe}$ is a perfectly valid element of the hyper-power set.
2. *PCR5 redistribution* – When the two sensors produce conflicting evidence, the Proportional Conflict Redistribution rule number 5 (PCR5) takes each partial conflict (e.g., the 0.6×0.7 product) and sends that conflict mass back only to the two propositions that generated it, in proportion to the original masses. In practice:
 - Part of the 0.42 conflict is returned to Friend (from Sensor A) proportional to 0.6.
 - Part is returned to Foe (from Sensor B) proportional to 0.7.

The redistribution is done locally for every conflicting pair, and the empty set receives zero mass—there is no global normalization step.

3. *Resulting belief distribution* – After applying PCR5, the final combined belief still reflects a substantial amount of mass on Friend and Foe (because the conflict was allocated back to those very hypotheses) and only a modest increase on Neutral. This outcome respects the fact that the two sensors are genuinely at odds, preserving the “Friend-Foe paradox” instead of smoothing it away.

6.3 | Summary

- DST assumes that categories like Friend and Foe cannot overlap. When sensors disagree, the theory throws away the conflict and rescales the remaining numbers, which can misleadingly boost unrelated categories (e.g., Neutral).

- DSMT relaxes the exclusivity rule, allowing overlapping categories. Its PCR5 rule returns the conflicting evidence to the original, conflicting categories in proportion to how strongly each sensor supported them, yielding a more intuitive picture of disagreement.
- An important topic to display is the dictatorial behavior of the Dempster-Shafer (DS) rule. We know that even when the conflict is low, the DS rule can, in countless examples, completely mask the impact of a source on the fusion result. In our opinion, the problem of the unacceptable behavior of the DS rule should be mentioned in the article to justify the need for other, more advanced fusion techniques.
- The famous Zadeh Counter-Example shows that Dempster's Rule totally fails.

7 | DST Rule completely ignoring a non-vacuous mass

The below counter-example to DST Rule is taken from [8].

Consider a medical diagnosis problem with a frame of discernment $\Theta = \{A, B, C\}$, where A, B and C correspond to three distinct pathologies (say A = brain tumor, B = concussion and C = meningitis) of a patient that these pathologies do not occur simultaneously.

Two doctors give their own medical diagnostic for the same patient, based on their own knowledges after analyzing symptoms, IRM images, or any useful medical results in the form two non-Bayesian bba's $m_1(\cdot)$ and $m_2(\cdot)$ as follows:

$$\begin{aligned} m_1, m_2 : \Theta &\rightarrow [0,1] \\ m_1(A) &= a, m_1(A \cup B) = 1-a, m_1(C) = 0, m_1(A \cup B \cup C) = 0 \\ m_2(A) &= 0, m_2(A \cup B) = b_1, m_2(C) = 1-b_1-b_2, m_2(A \cup B \cup C) = b_2 \end{aligned}$$

The parameters a, b1, and b2 can take any value, as long as a is in [0, 1].

$$b_1, b_2 > 0$$

$$b_1 + b_2 \in [0,1]$$

Using the conjunctive operator, we have

$$\begin{aligned} m_{12}(A) &= a(b_1 + b_2) \\ m_{12}(A \cup B) &= (1-a)(b_1 + b_2) \end{aligned}$$

$$K_{12} = m_{12}(\phi) = 1 - b_1 - b_2 \text{ (the conflicting mass)}$$

Using DS rule after normalization by $1 - K_{12} = b_1 + b_2$, the result is:

$$\begin{aligned} m_{DS}(A) &= \frac{m_{12}(A)}{1 - K_{12}} = \frac{a(b_1 + b_2)}{b_1 + b_2} = a = m_1(A) \\ m_{DS}(A \cup B) &= \frac{m_{12}(A \cup B)}{1 - K_{12}} = \frac{(1-a)(b_1 + b_2)}{b_1 + b_2} = 1 - a = m_1(A \cup B) \end{aligned}$$

With Shafer's reasoning and DS rule, one sees that Doctor 2 report is actually ignored in the DS fusion process because one gets $m_{DS}(\cdot) = m_1(\cdot)$. This dictatorial behavior is counter-intuitive because Doctor 2 is not a total ignorant source of evidence and he brings also useful information. We see that the level of conflict $1-b_1-b_2$ between the two medical diagnostics doesn't matter in fact in the DS fusion process in this example, since it can be chosen at any high or low level, depending on the choice of b_1+b_2 .

8 | Concluding Remarks

DST remains a powerful, elegant tool when the problem domain naturally satisfies exclusivity and conflict stays modest. DS^mT expands the horizon to high-conflict, vague, or overlapping scenarios, offering a richer algebra (hyper-power set) and a more nuanced conflict redistribution (PCR rules). By recognizing the model assumptions and conflict handling philosophy behind each theory, practitioners can select the framework that aligns with the nature of their data and the fidelity required for decision-making. In many modern applications—multisensory fusion, intelligence analysis, and complex risk assessment—DS^mT provides the flexibility and robustness that DST alone cannot guarantee, while still encompassing DST as a special, low-conflict case. There also are many cases where Dempster's Rule completely fails, so it is unreliable in practical applications.

Author Contribution

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data, but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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