



Plithgoenic Fuzzy Risk Management and Plithgoenic Fuzzy IT Service Management

Takaaki Fujita^{1*} 

¹ Independent Researcher, Tokyo, Japan; takaaki.fujita060@gmail.com.

Abstract: Plithogenic sets describe each element by facetwise (attribute-driven) membership degrees and a contradiction function on the facet domain. Upside-Down Logic provides a disciplined transformation that interchanges truth and falsity by systematically inverting evaluations while maintaining coherence of the underlying context. Risk management concerns the identification, assessment, and mitigation of potential losses so that organizational objectives are less affected. IT service management (ITSM) concerns the design, delivery, operation, and continual improvement of IT services so that business needs are met effectively, efficiently, and consistently.

In this paper we introduce the notions of *Plithgoenic Fuzzy Risk Management* and *Plithgoenic Fuzzy IT Service Management* and analyze their basic structural properties. In addition, using the viewpoint of *U-Sets* and *U-Structures*, we briefly discuss uncertainty-domain extensions, namely *Uncertain Risk Management* and *Uncertain IT Service Management*.

Keywords: Plithgoenic Fuzzy Risk Management, Plithgoenic Fuzzy IT Service Management, Fuzzy Set, Uncertain Set, Plithogenic Set, Upside-down logic

1 Preliminaries

This section fixes notation and collects the background material that will be used in the statements and proofs of the main results.

1.1 Upside-Down Logic

We adopt the notion of *Upside-Down Logic*: after an appropriate transformation of the surrounding context (and, if needed, of the lemma itself), statements that are true become false and statements that are false become true. This mechanism is intended to model reversals and context-dependent ambiguity in reasoning systems; see, for instance, [1, 2, 3, 4].

Definition 1 (Context). [2, 3] A *context* $\mathcal{C}$ is a collection of parameters or conditions relative to which lemmas are evaluated; typical parameters may be spatial, temporal, semantic, or interpretive.

Definition 2 (Logical system). (cf. [5]) A *logical system* is a structure

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where $\mathcal{P}$ is the set of lemmas (formulas) in a formal language $\mathcal{L}$, $\mathcal{V}$ is a set of truth values (e.g., {True, False} in the classical case), and $v : \mathcal{P} \rightarrow \mathcal{V}$ is a valuation assigning to each lemma a truth value. Optionally, $\mathcal{M}$ may also specify a set of axioms $\mathcal{A} \subseteq \mathcal{P}$ and a collection of inference rules $\mathcal{I}$.

Notation 3. Let $\mathcal{P}$ be a set of lemmas and let $\mathcal{C}$ be a family of contexts. We write

$$T : \mathcal{P} \times \mathcal{C} \longrightarrow \{\text{True}, \text{False}, \text{Indeterminate}\}$$

for a context-dependent evaluation map that assigns a status to each lemma–context pair. We also fix a formal language $\mathcal{L}$ and a logical system $\mathcal{M} = (\mathcal{P}, \mathcal{V}, v)$ as above.

Definition 4 (Upside-Down Logic). [2, 3] An *Upside-Down Logic* is obtained by extending a logical system $\mathcal{M}$ with a transformation U acting on lemmas and/or contexts, thereby producing a system $\mathcal{M}'$ such that for every lemma $A \in \mathcal{P}$ and every context $\mathcal{C}$:

- (Truth $\rightarrow$ falsity) If $v(A) = \text{True}$ in $\mathcal{C}$, then $v(U(A)) = \text{False}$ in $U(\mathcal{C})$.
- (Falsity $\rightarrow$ truth) If $v(A) = \text{False}$ in $\mathcal{C}$, then $v(U(A)) = \text{True}$ in $U(\mathcal{C})$.

In addition, U is required to be well posed and internally consistent within $\mathcal{M}'$.

Example 5 (Threshold/comparator flip without changing the context). Assume the language contains atomic statements

$$A_\theta : (x \geq \theta) \quad (\theta \in \mathbb{R}),$$

and take the context to be the parameter value $\mathcal{C}_\theta := \{\theta\}$. Fix a valuation v so that $v(A_\theta) = \text{True}$ in $\mathcal{C}_\theta$ precisely when the designated real number x satisfies $x \geq \theta$.

Define an Upside-Down transform U by

$$U(A_\theta) := (x < \theta), \quad U(\mathcal{C}_\theta) := \mathcal{C}_\theta.$$

Then:

- If $v(A_\theta) = \text{True}$ in $\mathcal{C}_\theta$, then $x \geq \theta$, hence $x < \theta$ is false in the same context, i.e., $v(U(A_\theta)) = \text{False}$ in $U(\mathcal{C}_\theta)$.
- If $v(A_\theta) = \text{False}$ in $\mathcal{C}_\theta$, then $x < \theta$, so $v(U(A_\theta)) = \text{True}$ in $U(\mathcal{C}_\theta)$.

Moreover, U is involutive on these formulas:

$$U(U(A_\theta)) = U(x < \theta) = (x \geq \theta) = A_\theta,$$

so adding U yields a well posed and internally consistent extension $\mathcal{M}'$.

1.2 Plithogenic Set

Plithogenic sets[6, 7] encode membership through explicit attributes together with a contradiction mapping between attribute values. In this way they subsume and extend several degree-based frameworks, including fuzzy [8], intuitionistic [9], hesitant fuzzy sets [10], and neutrosophic sets [11, 12].

Definition 6 (Fuzzy Set). [8] Let Y be a nonempty universe. A *fuzzy set* on Y is a mapping $\tau : Y \rightarrow [0, 1]$, and a *fuzzy relation* on Y is a mapping $\delta : Y \times Y \rightarrow [0, 1]$. If τ is a fuzzy set on Y , then δ is called a *fuzzy relation on* τ if, for all $y, z \in Y$,

$$\delta(y, z) \leq \min\{\tau(y), \tau(z)\}.$$

Definition 7 (Plithogenic Set). [13] Let S be a universe and let $P \subseteq S$ be a nonempty subset. A *plithogenic set* is a 5-tuple

$$PS = (P, v, Pv, pdf, pCF),$$

consisting of:

- an attribute v ;
- its value domain Pv ;
- a *degree of appurtenance* map (DAF)

$$pdf : P \times Pv \rightarrow [0, 1]^s;$$

- a *degree of contradiction* map (DCF)

$$pCF : Pv \times Pv \rightarrow [0, 1]^t.$$

For all $a, b \in Pv$, the DCF satisfies

$$pCF(a, a) = 0 \quad \text{and} \quad pCF(a, b) = pCF(b, a).$$

Here $s, t \in \mathbb{N}$ denote, respectively, the appurtenance and contradiction dimensions.

Definition 8 (Plithogenic Fuzzy Set ($s = 1, t = 1$)). [14, 15] A *plithogenic fuzzy set* is a plithogenic set $PS = (P, v, Pv, pdf, pCF)$ for which

$$pdf : P \times Pv \rightarrow [0, 1], \quad pCF : Pv \times Pv \rightarrow [0, 1].$$

For $x \in P$ and $a \in Pv$ we write

$$\mu_P(x | a) := pdf(x, a) \in [0, 1]$$

for the (scalar) membership of x under the attribute value a . The contradiction between attribute values is the scalar

$$c(a, b) := pCF(a, b) \in [0, 1], \quad c(a, a) = 0, \quad c(a, b) = c(b, a).$$

.

Example 9 (Concrete plithogenic fuzzy set ($s = t = 1$)). Let $P = \{s_{\text{web}}, s_{\text{email}}, s_{\text{storage}}\}$ be three IT services, and take the attribute $v = \text{“evaluation facet”}$ with

$$Pv = \{Q, U, R\},$$

standing for Quality, Utility, and Reliability. Since $s = t = 1$, both DAF and DCF are scalar.

Memberships. Define $pdf : P \times Pv \rightarrow [0, 1]$ by

	Q	U	R
s_{web}	0.82	0.76	0.91
s_{email}	0.75	0.88	0.86
s_{storage}	0.80	0.72	0.94

so $\mu_P(s_{\text{web}} | R) = 0.91$.

Contradiction. Let $c(a, b) := pCF(a, b) \in [0, 1]$ be given by the symmetric, reflexive-zero matrix

$$(c(a, b))_{a, b \in Pv} = \begin{pmatrix} 0 & 0.60 & 0.30 \\ 0.60 & 0 & 0.50 \\ 0.30 & 0.50 & 0 \end{pmatrix}.$$

Then $c(Q, Q) = 0$ and $c(U, R) = c(R, U) = 0.50$, etc. Therefore $PS = (P, v, Pv, pdf, pCF)$ is a plithogenic fuzzy set with scalar membership and scalar contradiction.

For reference, Table 1 presents a comparison between a fuzzy set and a plithogenic fuzzy set ($s = t = 1$). A fuzzy set assigns a single membership degree to each element, whereas a plithogenic fuzzy set provides facet-indexed memberships together with an explicit contradiction map over attribute values.

1.3 Upside-Down Logic in Plithogenic Fuzzy Set with Contradiction Reset

We introduce an *upside-down* mechanism for plithogenic fuzzy sets in which certain membership grades are complemented whenever their contradiction with a fixed anchor exceeds a specified threshold. After performing such a flip, the contradiction value responsible for the activation is immediately reset to zero, which prevents the same trigger from firing again at the same threshold and thereby stabilizes the context (cf. [16, 4]).

TABLE 1. Concise comparison of a fuzzy set and a plithogenic fuzzy set ($s = t = 1$).

Aspect	Fuzzy set	Plithogenic fuzzy set ($s = t = 1$)
Universe / support	Universe Y (elements $y \in Y$).	Universe S with a nonempty support subset $P \subseteq S$ (elements $x \in P$).
Primary data	A single membership function $\tau : Y \rightarrow [0, 1]$.	An attribute v with value-domain Pv , plus two maps: $pdf : P \times Pv \rightarrow [0, 1]$ and $pCF : Pv \times Pv \rightarrow [0, 1]$.
Meaning of membership	$\tau(y)$ is the degree to which y belongs to the set.	$pdf(x, a) =: \mu_P(x \mid a)$ is the degree to which x belongs <i>under facet/value</i> $a \in Pv$.
Context / facets	No explicit attribute-value layer; membership is facet-free.	Explicit facet layer: each $a \in Pv$ provides a separate membership view.
Contradiction / conflict	No built-in contradiction measure between evaluation criteria.	A contradiction map $c(a, b) =: pCF(a, b)$ quantifies conflict between facets and satisfies $c(a, a) = 0$ and $c(a, b) = c(b, a)$.
Relations	A fuzzy relation is $\delta : Y \times Y \rightarrow [0, 1]$ constrained by $\delta(y, z) \leq \min\{\tau(y), \tau(z)\}$.	The defining structure is not a relation on P ; rather, it is the pair (pdf, pCF) over (P, Pv) , enabling contradiction-aware aggregation across facets.
Reduction / embedding	—	Any fuzzy set $\tau : P \rightarrow [0, 1]$ is recovered as a special case by taking $Pv = \{a_0\}$ and setting $pdf(x, a_0) = \tau(x)$ (with necessarily $pCF(a_0, a_0) = 0$).

Definition 10 (Upside-down transform with contradiction reset for a Plithogenic Fuzzy Set). (cf. [16, 4]) Let $PS = (P, v, Pv, pdf, pCF)$ be a plithogenic fuzzy set. Write

$$\mu_P(x \mid a) := pdf(x, a) \in [0, 1], \quad c(a, b) := pCF(a, b) \in [0, 1],$$

where $c(a, a) = 0$ and $c(a, b) = c(b, a)$ for all $a, b \in Pv$. Fix an anchor value $b \in Pv$ and a threshold $\tau \in [0, 1]$, and define the activation locus

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The *upside-down transform with contradiction reset* produces a new plithogenic fuzzy set

$$PS^{U_{b,\tau}} := (P, v, Pv, pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}})$$

defined as follows. For every $x \in P$ and $a \in Pv$,

$$pdf^{U_{b,\tau}}(x, a) := \begin{cases} 1 - \mu_P(x \mid a), & \text{if } a \in A_\tau(b), \\ \mu_P(x \mid a), & \text{if } a \notin A_\tau(b). \end{cases}$$

Moreover, the updated contradiction function $pCF^{U_{b,\tau}} : Pv \times Pv \rightarrow [0, 1]$ is given, for all $u, v \in Pv$, by

$$pCF^{U_{b,\tau}}(u, v) := \begin{cases} 0, & \text{if } \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

In words: whenever a facet a is flipped because its contradiction with the anchor b meets (or exceeds) τ , the pairwise contradiction between a and b is subsequently forced to 0, so that the same flip cannot be instantly reactivated at the same threshold.

Example 11 (A concrete Upside-Down transform with contradiction reset). Let $PS = (P, v, Pv, pdf, pCF)$ be as in Definition 10. Consider

$$P = \{x_1, x_2, x_3\}, \quad Pv = \{A, B, C\},$$

where A, B, C represent three evaluation facets. The initial memberships $\mu_P(x \mid a) = pdf(x, a)$ are

	A	B	C
x_1	0.20	0.60	0.70
x_2	0.90	0.40	0.50
x_3	0.30	0.80	0.20

and the symmetric contradiction values $c(\cdot, \cdot) = pCF(\cdot, \cdot)$ are

$$(c(a, b))_{a, b \in \{A, B, C\}} = \begin{pmatrix} 0 & 0.30 & 0.80 \\ 0.30 & 0 & 0.40 \\ 0.80 & 0.40 & 0 \end{pmatrix}.$$

Choose anchor $b = C$ and threshold $\tau = 0.7$. Then

$$A_\tau(C) = \{a \in \{A, B, C\} : c(a, C) \geq 0.7\} = \{A\},$$

because $c(A, C) = 0.80$ while $c(B, C) = 0.40$ and $c(C, C) = 0$.

Membership update. Only the facet A is activated, so exactly the A -column is complemented:

$$pdf^{U_{C,0.7}}(x, a) = \begin{cases} 1 - \mu_P(x | A), & a = A, \\ \mu_P(x | a), & a \in \{B, C\}. \end{cases}$$

Hence the transformed memberships are

	A	B	C
x_1	$1 - 0.20 = 0.80$	0.60	0.70
x_2	$1 - 0.90 = 0.10$	0.40	0.50
x_3	$1 - 0.30 = 0.70$	0.80	0.20

Contradiction reset. Because $\{A, C\}$ is the unique activated pair, we reset that contradiction to 0 and keep all other entries unchanged:

$$pCF^{U_{C,0.7}}(u, v) = \begin{cases} 0, & \{u, v\} = \{A, C\}, \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

Therefore,

$$(c^{U_{C,0.7}}(a, b))_{a, b \in \{A, B, C\}} = \begin{pmatrix} 0 & 0.30 & 0 \\ 0.30 & 0 & 0.40 \\ 0 & 0.40 & 0 \end{pmatrix}.$$

The output still satisfies $pdf^{U_{C,0.7}}(\cdot, \cdot) \in [0, 1]$, and the updated contradiction map remains symmetric with zero diagonal. Furthermore, since $c^{U_{C,0.7}}(A, C) = 0 < \tau$, the specific flip just applied cannot be immediately triggered again at the same threshold, which illustrates the stabilizing role of the reset.

1.4 Fuzzy Risk Management

Risk management concerns identifying, assessing, and reducing potential losses so that their impact on organizational goals is minimized [17, 18]. The use of fuzzy logic in risk-management formulations has been studied widely; see, e.g., [19, 20].

Definition 12 (Mathematical framework for Fuzzy Risk Management). [21] Let $(\Omega, \mathcal{F}, P)$ be a probability space, and let $D \subseteq \mathbb{R}^n$ be a nonempty closed convex decision set. Let

$$L : D \longrightarrow L^\infty(\Omega, \mathcal{F}, P), \quad x \longmapsto L(x),$$

assign to each decision $x \in D$ an (essentially bounded) loss random variable $L(x)$. Let

$$\rho : L^\infty(\Omega, \mathcal{F}, P) \longrightarrow \mathbb{R}$$

be a (coherent) risk measure in the crisp setting. Finally, let

$$\Phi : \mathbb{R} \longrightarrow [0, 1]$$

be a continuous and strictly decreasing “satisfaction–risk” mapping (for example, $\Phi(r) = e^{-\lambda r}$ for some $\lambda > 0$). Define the *fuzzy decision set*

$$\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x)) \mid x \in D\},$$

where the membership function $\mu_{\mathcal{A}} : D \rightarrow [0, 1]$ is

$$\mu_{\mathcal{A}}(x) = \Phi(\rho(L(x))).$$

The *fuzzy risk management problem* is to choose

$$x^* \in \arg \max_{x \in D} \mu_{\mathcal{A}}(x),$$

which is equivalent to $\min_{x \in D} \rho(L(x))$ while expressing gradual preference through Φ .

1.5 Fuzzy IT Service Management

IT Service Management (ITSM) refers to the systematic design, delivery, operation, and continual improvement of IT services so that business requirements are met effectively, efficiently, and consistently [22, 23]. In practice, widely adopted guidance includes ITIL (Information Technology Infrastructure Library) (cf. [24, 25, 26]) and SIAM (Service Integration and Management) (cf. [27, 28]). Below we recall a mathematical formulation of a *Fuzzy IT Service Management* framework, which extends ITSM by embedding fuzzy uncertainty into core evaluation components.

Definition 13 (Mathematical Framework for Fuzzy IT Service Management). [29] The *Fuzzy IT Service Management* (FITSM) framework augments a conventional ITSM model by allowing uncertainty in key assessments (such as quality, utility/cost, and reliability) to be represented by fuzzy values. Formally, an FITSM system is described by a tuple

$$\mathcal{FITSM} = (S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi),$$

where:

- **S**: a finite set of IT services offered by an organization (e.g., $S = \{s_{\text{email}}, s_{\text{web}}, s_{\text{storage}}\}$).
- **I**: a finite set of infrastructure components (servers, routers, storage devices, etc.). Each $i \in I$ has associated attributes:
 - reliability $r(i) \in [0, 1]$, interpreted as the probability that i operates without failure;
 - operating cost $c(i) \in \mathbb{R}^+$, interpreted as the cost of running or maintaining i .
- **P**: a finite set of IT management processes (e.g., Incident Management, Change Management, Problem Management). Process compliance with service requirements is represented by a mapping

$$\psi : P \times S \rightarrow \{0, 1\},$$

where $\psi(p, s) = 1$ indicates that process p satisfies the requirements for service s .

- **M**: an allocation mapping assigning to each service the supporting infrastructure and associated processes:

$$M : S \rightarrow 2^I \times 2^P.$$

If $M(s) = (I_s, P_s)$, then $I_s \subseteq I$ is the set of components supporting s , and $P_s \subseteq P$ is the set of processes associated with s .

- **C**: a collection of cost parameters (for example, overall budget constraints or cost multipliers attached to particular components or processes).
- φ : an aggregation function used to combine component reliabilities/availabilities; for instance, $\varphi : 2^I \rightarrow [0, 1]$ may aggregate the reliabilities of the components assigned to a service.
- $\tilde{Q}$: a *fuzzy quality function*

$$\tilde{Q} : S \rightarrow \tilde{P}(\mathbb{R}^+),$$

assigning to each service a fuzzy number (or a fuzzy set of performance scores) to reflect uncertainty in quality measurement; for example, a quality evaluation may be expressed as $\{200.65, 201.00, 200.30\}$ rather than a single scalar.

- $\tilde{U}$: a *fuzzy utility function*

$$\tilde{U} : S \rightarrow \tilde{P}(\mathbb{R}),$$

capturing uncertainty in economic evaluation, such as variability in expected revenue or fluctuations in operational costs.

- $\tilde{R}$: a fuzzy reliability function

$$\tilde{R} : S \rightarrow \tilde{P}([0, 1]),$$

assigning to each service a fuzzy representation of overall reliability.

In this formulation, the fuzzy maps $\tilde{Q}$, $\tilde{U}$, and $\tilde{R}$ assign to each service a *set* of plausible values (or a fuzzy number) rather than a single crisp value. This choice models real-world ambiguity in IT service evaluation and, in particular, endows the FITSM framework with the characteristic structure of a fuzzy set.

2 Main Results

This section states the principal contributions of the paper and provides the corresponding explanations.

2.1 Plithogenic Fuzzy Risk Management

We incorporate several *risk facets* (attribute values) together with an explicit *contradiction* among facets. Each facet yields a fuzzy acceptance level, and these facetwise acceptances are fused into a single overall acceptance by a plithogenic aggregation rule whose behavior is controlled by the contradiction degrees.

Definition 14 (Plithogenic Fuzzy Risk Management). A *Plithogenic Fuzzy Risk Management (PFRM) instance* is a tuple

$$\text{PFRM} := (D, v, Pv, pdf, pCF, \text{Agg}),$$

consisting of the following data:

- $D \subseteq \mathbb{R}^n$ is a nonempty set of decisions.
- v is the fixed attribute “risk facet” with value domain Pv (finite or countable). Elements $a \in Pv$ represent facets such as likelihood, impact, detectability, regulatory exposure, scenario choice, stakeholder viewpoint, and so on.
- For every facet $a \in Pv$ we are given:
 - a loss mapping $L_a : D \rightarrow L^\infty(\Omega, \mathcal{F}, \mathbb{P})$,
 - a (coherent) risk measure $\rho_a : L^\infty(\Omega, \mathcal{F}, \mathbb{P}) \rightarrow \mathbb{R}$,
 - a continuous and strictly decreasing satisfaction map $\Phi_a : \mathbb{R} \rightarrow [0, 1]$.

The induced *facetwise fuzzy acceptance* (plithogenic appurtenance) is

$$pdf(x, a) := \Phi_a(\rho_a(L_a(x))) \in [0, 1], \quad (x, a) \in D \times Pv.$$

- $pCF : Pv \times Pv \rightarrow [0, 1]$ is a *contradiction function* satisfying

$$pCF(a, b) = pCF(b, a), \quad pCF(a, a) = 0 \quad (a, b \in Pv).$$

Intuitively, $pCF(a, b)$ measures the degree of “tension” or “conflict” between facets a and b .

- Agg is a *plithogenic aggregation rule* producing an overall membership $\bar{\mu}(x) \in [0, 1]$ from the family $\{pdf(x, a)\}_{a \in Pv}$ while using pCF to interpolate between a fixed t -norm $T : [0, 1]^2 \rightarrow [0, 1]$ and a fixed t -conorm $S : [0, 1]^2 \rightarrow [0, 1]$.¹ For two facets $a, b \in Pv$ we define the contradiction-modulated binary operator

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b))T(p, q) + pCF(a, b)S(p, q), \quad p, q \in [0, 1],$$

and for $|Pv| > 2$ we extend by any associative fold (e.g., a fixed order or a rule prioritizing a designated facet). The resulting overall acceptance is

$$\bar{\mu}(x) := \text{Agg}(\{pdf(x, a)\}_{a \in Pv}, pCF) \in [0, 1].$$

Given PFRM, the *plithogenic fuzzy risk management problem* is

$$x^* \in \arg \max_{x \in D} \bar{\mu}(x).$$

¹Common choices include $T(p, q) = \min\{p, q\}$ or pq , and $S(p, q) = \max\{p, q\}$ or $p + q - pq$.

Example 15 (Mitigation Strategy Selection with Three Risk Facets). *Decisions and facets.* Let $D = \{x_A, x_B, x_C\}$ represent three candidate mitigation strategies. Take three facets $Pv = \{L, I, D\}$, where L = likelihood, I = impact, and D = detectability (a larger detectability score means poorer detectability).

Facetwise loss, risk measure, and satisfaction. For each facet $a \in Pv$ and decision $x \in D$, let $L_a(x)$ be a bounded loss random variable. In this simplified illustration, each $L_a(x)$ is constant and equals a score $r_{x,a} \in [0, 10]$. We set ρ_a to be the identity and take the satisfaction map

$$\Phi_a(r) := \max\{0, \min\{1, 1 - r/10\}\},$$

which decreases linearly with the score. Hence the facetwise acceptance is

$$pdf(x, a) = \Phi_a(\rho_a(L_a(x))) = 1 - \frac{r_{x,a}}{10} \in [0, 1].$$

Write $\mu(x | a) := pdf(x, a)$.

Facet scores and induced memberships.

	$r_{x,L}$	$r_{x,I}$	$r_{x,D}$	$\Rightarrow$		$\mu(x L)$	$\mu(x I)$	$\mu(x D)$
x_A	4	6	3		x_A	0.6	0.4	0.7
x_B	5	5	6		x_B	0.5	0.5	0.4
x_C	3	7	5		x_C	0.7	0.3	0.5

Contradiction map. Let $pCF : Pv \times Pv \rightarrow [0, 1]$ be symmetric with $pCF(a, a) = 0$. Set

$$pCF(L, I) = 0.5, \quad pCF(I, D) = 0.3, \quad pCF(L, D) = 0.2,$$

and extend by symmetry.

Plithogenic aggregation. Choose the t -norm $T(p, q) = pq$ and the t -conorm $S(p, q) = p + q - pq$. For facets a, b define

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b))T(p, q) + pCF(a, b)S(p, q).$$

We aggregate in the fixed order $\pi = (L, I, D)$ by

$$\begin{aligned} \bar{\mu}_1(x) &:= \mu(x | L), & \bar{\mu}_2(x) &:= T_{pCF}(\bar{\mu}_1(x), \mu(x | I); L, I), \\ \bar{\mu}_3(x) &:= T_{pCF}(\bar{\mu}_2(x), \mu(x | D); I, D), & \bar{\mu}(x) &:= \bar{\mu}_3(x). \end{aligned}$$

Step-by-step computation (explicit arithmetic). Using $c = pCF(a, b)$,

$$T_{pCF}(p, q; a, b) = (1 - c)pq + c(p + q - pq) = pq + cp + cq - 2cpq.$$

x_A : $\mu_L = 0.6, \mu_I = 0.4, \mu_D = 0.7$.

$$\bar{\mu}_2 = T_{pCF}(0.6, 0.4; L, I), \quad c = 0.5 : pq = 0.24, \quad cp = 0.30, \quad cq = 0.20, \quad 2cpq = 0.24 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.70; I, D), \quad c = 0.3 : pq = 0.35, \quad cp = 0.15, \quad cq = 0.21, \quad 2cpq = 0.21 \Rightarrow \bar{\mu}(x_A) = 0.50.$$

x_B : $\mu_L = 0.5, \mu_I = 0.5, \mu_D = 0.4$.

$$\bar{\mu}_2 = T_{pCF}(0.5, 0.5; L, I), \quad c = 0.5 : pq = 0.25, \quad cp = 0.25, \quad cq = 0.25, \quad 2cpq = 0.25 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.40; I, D), \quad c = 0.3 : pq = 0.20, \quad cp = 0.15, \quad cq = 0.12, \quad 2cpq = 0.12 \Rightarrow \bar{\mu}(x_B) = 0.35.$$

x_C : $\mu_L = 0.7, \mu_I = 0.3, \mu_D = 0.5$.

$$\bar{\mu}_2 = T_{pCF}(0.7, 0.3; L, I), \quad c = 0.5 : pq = 0.21, \quad cp = 0.35, \quad cq = 0.15, \quad 2cpq = 0.21 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.50; I, D), \quad c = 0.3 : pq = 0.25, \quad cp = 0.15, \quad cq = 0.15, \quad 2cpq = 0.15 \Rightarrow \bar{\mu}(x_C) = 0.40.$$

Decision. We obtain

$$\bar{\mu}(x_A) = 0.50, \quad \bar{\mu}(x_C) = 0.40, \quad \bar{\mu}(x_B) = 0.35,$$

so the PFRM rule selects x_A as the preferred mitigation strategy.

Remark 16 (Design freedom and well-posedness). *The scheme is flexible while remaining mathematically well-defined: (i) $\text{pdf}(x, a) \in [0, 1]$ holds by construction from Φ_a and ρ_a ; (ii) for $c \in [0, 1]$ the mixture $(1-c)T + cS$ stays in $[0, 1]$ whenever T and S are standard t -(co)norms; hence (iii) $\bar{\mu} : D \rightarrow [0, 1]$ is well-defined. Under standard compactness/continuity hypotheses on D and on $x \mapsto \text{pdf}(x, a)$, an optimizer exists.*

Theorem 17 (Reduction to classical FRM). *Let $\text{PFRM} = (D, v, Pv, \text{pdf}, \text{pCF}, \text{Agg})$ be as in Definition 14. Assume that $Pv = \{a_0\}$ and $\text{pCF} \equiv 0$. Then, for every $x \in D$,*

$$\bar{\mu}(x) = \text{pdf}(x, a_0) = \Phi_{a_0}(\rho_{a_0}(L_{a_0}(x))),$$

and maximizing $\bar{\mu}$ over D is identical to the classical FRM problem with $(L, \rho, \Phi) = (L_{a_0}, \rho_{a_0}, \Phi_{a_0})$.

Proof: When Pv has exactly one element a_0 , the aggregation rule receives only the single value $\text{pdf}(x, a_0)$, so by the definition of an associative fold we have $\bar{\mu}(x) = \text{pdf}(x, a_0)$. Since $\text{pCF}(a_0, a_0) = 0$, the contradiction-based modulation does not affect anything. Substituting the definition of pdf yields the displayed identity, and the two maximization problems coincide. $\square$

Theorem 18 (Plithogenic fuzzy set structure). *For any $\text{PFRM} = (D, v, Pv, \text{pdf}, \text{pCF}, \text{Agg})$ as in Definition 14, the tuple*

$$PS := (P, v, Pv, \text{pdf}, \text{pCF}), \quad P := D,$$

is a plithogenic fuzzy set (scalar appurtenance and scalar contradiction), namely:

- (a) $\text{pdf} : P \times Pv \rightarrow [0, 1]$ is well-defined via $\text{pdf}(x, a) = \Phi_a(\rho_a(L_a(x)))$,
- (b) $\text{pCF} : Pv \times Pv \rightarrow [0, 1]$ is symmetric and satisfies $\text{pCF}(a, a) = 0$.

Proof: (a) For each $a \in Pv$ and $x \in P = D$, we have $L_a(x) \in L^\infty$, hence $\rho_a(L_a(x)) \in \mathbb{R}$, and therefore $\Phi_a(\rho_a(L_a(x))) \in [0, 1]$ because $\Phi_a : \mathbb{R} \rightarrow [0, 1]$. Thus $\text{pdf}(x, a) \in [0, 1]$ for all $(x, a) \in P \times Pv$.

(b) The axioms $\text{pCF}(a, b) = \text{pCF}(b, a)$ and $\text{pCF}(a, a) = 0$ are assumed in the definition of PFRM . These are precisely the required conditions for the contradiction function in a plithogenic fuzzy set. Hence $(P, v, Pv, \text{pdf}, \text{pCF})$ is a plithogenic fuzzy set. $\square$

2.2 Upside-Down Logic in Plithogenic Fuzzy Risk Management

We now enrich the Plithogenic Fuzzy Risk Management (PFRM) framework from Definition 14 with an explicit and fully formal *Upside-Down* operator. The operator is triggered by strong contradiction against a chosen anchor facet: whenever a facet lies in sufficiently large contradiction with the anchor, its facetwise acceptability is complemented. We then verify that, when elementary acceptability statements are evaluated at the mid-cut, this operator implements the characteristic Upside-Down Logic behavior (truth and falsity are interchanged) on the activated facets.

Notation 19 (Facetwise memberships and contradiction). Let $\text{PFRM} = (D, v, Pv, \text{pdf}, \text{pCF}, \text{Agg})$ be a PFRM instance. We denote

$$\mu(x \mid a) := \text{pdf}(x, a) \in [0, 1] \quad (x \in D, a \in Pv),$$

and we write

$$c(a, b) := \text{pCF}(a, b) \in [0, 1] \quad (a, b \in Pv),$$

so that $c(a, a) = 0$ and $c(a, b) = c(b, a)$ for all $a, b \in Pv$.

Definition 20 (Anchor-threshold activated Upside-Down operator on PFRM). Fix an *anchor facet* $b \in Pv$ and a *contradiction threshold* $\tau \in [0, 1]$. Define the activation locus

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The associated *Upside-Down transform* $U_{b, \tau}$ maps the instance PFRM to a new instance

$$\text{PFRM}^{U_{b, \tau}} := (D, v, Pv, \text{pdf}^{U_{b, \tau}}, \text{pCF}^{U_{b, \tau}}, \text{Agg}^{U_{b, \tau}}),$$

where, for every $x \in D$ and $a \in Pv$,

$$pdf^{U_{b,\tau}}(x, a) := \begin{cases} 1 - \mu(x \mid a), & a \in A_\tau(b), \\ \mu(x \mid a), & a \notin A_\tau(b), \end{cases}$$

$$pCF^{U_{b,\tau}}(u, v) := \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

Moreover, $\text{Agg}^{U_{b,\tau}}$ is obtained from Agg by substituting $pdf^{U_{b,\tau}}$ and $pCF^{U_{b,\tau}}$ for pdf and pCF , respectively, while keeping the underlying t -norm/ t -conorm and the folding policy unchanged. In words: high contradiction against the anchor complements the corresponding facetwise membership, and the specific anchor–facet contradiction is then *reset* to 0 to stabilize the post-transform context.

Example 21 (Regulated medical-device go-to-market risk). *Decisions and facets.* Let $D = \{x_A, x_B\}$ be two launch plans for a regulated medical device. Let

$$Pv = \{S, L, D, C, R\}$$

denote the facets Safety impact (S), Likelihood (L), Detectability (D), Cost pressure (C), and Regulatory exposure (R). Suppose the facetwise acceptances are

Plan	$\mu(\cdot \mid S)$	$\mu(\cdot \mid L)$	$\mu(\cdot \mid D)$	$\mu(\cdot \mid C)$	$\mu(\cdot \mid R)$
x_A	0.92	0.60	0.70	0.80	0.75
x_B	0.85	0.70	0.60	0.90	0.60

Contradiction and aggregation. Assume pCF is symmetric and satisfies $pCF(a, a) = 0$. The entries needed below are

$$pCF(S, L) = 0.30, \quad pCF(L, D) = 0.30, \quad pCF(D, C) = 0.40, \quad pCF(C, R) = 0.90.$$

Choose $T(p, q) = pq$ and $S(p, q) = p + q - pq$, and fold in the order $\pi = (S, L, D, C, R)$ using

$$T_{pCF}(p, q; a, b) = (1 - pCF(a, b))pq + pCF(a, b)(p + q - pq).$$

Overall memberships before the transform. For x_A one obtains

$$\begin{aligned} \bar{\mu}_1 &= 0.92, \\ \bar{\mu}_2 &= T_{pCF}(0.92, 0.60; S, L) = 0.676800, \\ \bar{\mu}_3 &= T_{pCF}(0.676800, 0.70; L, D) = 0.602544, \\ \bar{\mu}_4 &= T_{pCF}(0.602544, 0.80; D, C) = 0.65742464, \\ \bar{\mu}_5 &= T_{pCF}(0.65742464, 0.75; C, R) = 0.872227392, \end{aligned}$$

so $\bar{\mu}(x_A) = 0.872227392$. Likewise,

$$\bar{\mu}(x_B) = 0.827523432.$$

Hence, prior to any Upside–Down transformation, x_A is preferred.

Anchor–threshold Upside–Down. Let the anchor be R (regulatory exposure) and set $\tau = 0.8$. Then

$$A_\tau(R) = \{C\},$$

because $pCF(C, R) = 0.90 \geq \tau$. Applying $U_{R,\tau}$ gives

$$\mu^U(x \mid C) = 1 - \mu(x \mid C), \quad pCF^U(C, R) = 0.$$

In particular,

$$\mu^U(x_A \mid C) = 1 - 0.80 = 0.20, \quad \mu^U(x_B \mid C) = 1 - 0.90 = 0.10.$$

Overall memberships after the transform. Refolding in the same order (only the C-column and the pair (C, R) differ) yields

$$\bar{\mu}^U(x_A) = 0.258839520, \quad \bar{\mu}^U(x_B) = 0.165024240.$$

Thus the transformation strongly penalizes cost-driven acceptance under strict regulatory anchoring, while the ranking $x_A \succ x_B$ is preserved (with a larger separation).

Example 22 (Hospital disaster-recovery strategy selection). *Decisions and facets.* Let $D = \{x_{AA}, x_{CB}\}$ represent two DR strategies: Active–Active (x_{AA}) versus Cold Backup (x_{CB}). Use facets

$$Pv = \{S, L, B, R\},$$

where S is safety-critical impact, L is likelihood, B is budget adequacy, and R is regulatory/contractual compliance. Assume the facetwise memberships are

Strategy	$\mu(\cdot \mid S)$	$\mu(\cdot \mid L)$	$\mu(\cdot \mid B)$	$\mu(\cdot \mid R)$
x_{AA}	0.90	0.85	0.20	0.60
x_{CB}	0.70	0.60	0.90	0.50

Contradiction and aggregation. For the fold order $\pi = (S, L, B, R)$, assume the relevant contradictions are

$$pCF(S, L) = 0.20, \quad pCF(L, B) = 0.50, \quad pCF(B, R) = 0.30, \quad pCF(S, B) = 0.80,$$

with symmetry and diagonal zeros as usual. Again take $T(p, q) = pq$ and $S(p, q) = p + q - pq$, and fold using T_{pCF} .

Overall memberships before the transform. A direct calculation gives

$$\bar{\mu}(x_{AA}) = 0.452430, \quad \bar{\mu}(x_{CB}) = 0.503000,$$

so, before anchoring, Cold Backup is preferred.

Anchor–threshold Upside–Down. Choose the anchor $b = S$ (patient safety) and $\tau = 0.75$. Then

$$A_\tau(S) = \{B\},$$

because $pCF(S, B) = 0.80 \geq \tau$. Applying $U_{S, \tau}$ yields

$$\mu^U(x \mid B) = 1 - \mu(x \mid B), \quad pCF^U(S, B) = 0,$$

hence

$$\mu^U(x_{AA} \mid B) = 1 - 0.20 = 0.80, \quad \mu^U(x_{CB} \mid B) = 1 - 0.90 = 0.10.$$

Overall memberships after the transform. Refolding with the flipped B values gives

$$\bar{\mu}^U(x_{AA}) = 0.614430, \quad \bar{\mu}^U(x_{CB}) = 0.303000.$$

Accordingly, after safety anchoring the preference reverses to Active–Active, reflecting a rule in which strong safety priority turns budget-favorable choices into low-acceptance ones when they are in strong conflict with safety.

Lemma 23 (Well-posedness). *PFRM $^{U_{b, \tau}}$ is again a valid PFRM instance. In particular: for all $(x, a) \in D \times Pv$, one has $pdf^{U_{b, \tau}}(x, a) \in [0, 1]$; the map $pCF^{U_{b, \tau}}$ remains symmetric and satisfies $pCF^{U_{b, \tau}}(a, a) = 0$; hence $\text{Agg}^{U_{b, \tau}}$ defines an overall membership $\bar{\mu}^{U_{b, \tau}} : D \rightarrow [0, 1]$.*

Proof: Because $\mu(x \mid a) \in [0, 1]$, its complement $1 - \mu(x \mid a)$ also lies in $[0, 1]$, so $pdf^{U_{b, \tau}}(x, a) \in [0, 1]$ for all (x, a) . The update rule for $pCF^{U_{b, \tau}}$ either keeps the original value $pCF(u, v)$ or replaces it by 0 on an unordered pair $\{a, b\}$; in either case, symmetry is preserved, and diagonal entries remain 0. Finally, $\text{Agg}^{U_{b, \tau}}$ uses the same t -norm/ t -conorm mixture and the same folding policy as Agg , and standard t -(co)norm mixtures map $[0, 1]^2$ to $[0, 1]$, so the folded output remains in $[0, 1]$. $\square$

Definition 24 (Elementary mid-cut acceptability statements). Let $\mathcal{L}$ be the collection of *elementary acceptance statements*

$$\text{Acc}_{1/2}(x, a) \quad \text{“decision } x \text{ is acceptable on facet } a \text{ at the mid-cut”,} \quad (x \in D, a \in Pv).$$

For a given PFRM instance PFRM, define the *mid-cut valuation* $v_{b,\tau}$ on $\mathcal{L}$ by

$$v_{b,\tau}(\text{Acc}_{1/2}(x, a)) := \begin{cases} \text{True}, & \mu(x \mid a) \geq \frac{1}{2}, \\ \text{False}, & \mu(x \mid a) < \frac{1}{2}. \end{cases}$$

After applying $U_{b,\tau}$, we evaluate the same statements using $\text{pdf}^{U_{b,\tau}}$, producing a post-transform valuation $v'_{b,\tau}$.

Theorem 25 (Upside–Down flip on elementary acceptances). *Let PFRM be a PFRM instance and fix (b, τ) . For every $x \in D$ and $a \in Pv$ with $\mu(x \mid a) \neq \frac{1}{2}$,*

$$\begin{aligned} a \in A_\tau(b) &\implies v'_{b,\tau}(\text{Acc}_{1/2}(x, a)) = \neg v_{b,\tau}(\text{Acc}_{1/2}(x, a)), \\ a \notin A_\tau(b) &\implies v'_{b,\tau}(\text{Acc}_{1/2}(x, a)) = v_{b,\tau}(\text{Acc}_{1/2}(x, a)). \end{aligned}$$

Proof: If $a \notin A_\tau(b)$ then $\text{pdf}^{U_{b,\tau}}(x, a) = \mu(x \mid a)$, so the inequality against $\frac{1}{2}$ is identical before and after the transform, and the valuation is unchanged.

Assume $a \in A_\tau(b)$. Then $\text{pdf}^{U_{b,\tau}}(x, a) = 1 - \mu(x \mid a)$. If $\mu(x \mid a) > \frac{1}{2}$, then $1 - \mu(x \mid a) < \frac{1}{2}$, so the statement $\text{Acc}_{1/2}(x, a)$ is True before the transform and False after it. If $\mu(x \mid a) < \frac{1}{2}$, then $1 - \mu(x \mid a) > \frac{1}{2}$, so the truth value switches in the opposite direction. The boundary case $\mu(x \mid a) = \frac{1}{2}$ is excluded by hypothesis, and hence the flip is strict on activated facets. $\square$

Corollary 26 (Realization of Upside–Down Logic). *Let the context be $\mathcal{C} = (b, \tau)$ and let the Upside–Down operator be $U(\text{PFRM}; \mathcal{C}) = \text{PFRM}^{U_{b,\tau}}$ on instances, with the language $\mathcal{L}$ fixed as above. Then for every lemma $A \in \mathcal{L}$ of the form $\text{Acc}_{1/2}(x, a)$ with $\mu(x \mid a) \neq \frac{1}{2}$,*

$$v_{\mathcal{C}}(A) = \text{True} \implies v_{U(\mathcal{C})}(A) = \text{False}, \quad v_{\mathcal{C}}(A) = \text{False} \implies v_{U(\mathcal{C})}(A) = \text{True},$$

whenever $a \in A_\tau(b)$, and $v_{U(\mathcal{C})}(A) = v_{\mathcal{C}}(A)$ whenever $a \notin A_\tau(b)$. Thus, on the activated facets, the operator implements the truth–falsity inversion axioms of Upside–Down Logic for the elementary mid-cut acceptability statements.

Proof: This follows directly from Theorem 25 by rewriting the statements in valuation form and distinguishing the cases $a \in A_\tau(b)$ and $a \notin A_\tau(b)$. $\square$

2.3 Plithogenic Fuzzy IT Service Management

We extend the FITSM framework by adding a plithogenic layer: services are assessed through multiple explicit *evaluation facets* (attribute values), and a *contradiction* function governs how the facetwise memberships are fused into a single overall score.

Definition 27 (Plithogenic Fuzzy IT Service Management). A *P–FITSM instance* is a tuple

$$\text{PFITSM} = (S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi; v, Pv, \text{pdf}, \text{pCF}, \text{Agg}),$$

where $(S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi)$ is an underlying FITSM system and, in addition, the following plithogenic components are specified:

- v is the attribute “service evaluation facet” with a (finite or countable) value domain Pv . A standard choice is $Pv = \{Q, U, R\}$ (Quality, Utility, Reliability), while further facets (e.g. security, compliance, sustainability) may also be included.

- $pdf : S \times Pv \rightarrow [0, 1]$ is the plithogenic appurtenance map, assigning a membership degree to each pair (s, a) by

$$pdf(s, a) = \begin{cases} \mu_Q(s), & a = Q, \\ \mu_U(s), & a = U, \\ \mu_R(s), & a = R, \\ F_a(\tilde{X}_a(s)), & \text{for any additional facet } a, \end{cases}$$

where each additional facet a is equipped with a fuzzy score $\tilde{X}_a : S \rightarrow \tilde{P}(\mathbb{R})$ and a monotone scoring functional $F_a : \tilde{P}(\mathbb{R}) \rightarrow [0, 1]$.

- $pCF : Pv \times Pv \rightarrow [0, 1]$ is the contradiction function, required to be symmetric and reflexive-zero:

$$pCF(a, b) = pCF(b, a), \quad pCF(a, a) = 0 \quad (a, b \in Pv).$$

Conceptually, $pCF(a, b)$ measures the degree of conflict between facets a and b .

- **Agg** is a plithogenic aggregator that combines the facetwise vector $\{pdf(s, a)\}_{a \in Pv}$ into an *overall service membership* $\bar{\mu}(s) \in [0, 1]$ by mixing a fixed t -norm T and a fixed t -conorm S according to contradiction. For two facets $a, b \in Pv$ and inputs $p, q \in [0, 1]$, set

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q).$$

When $|Pv| > 2$, fix a total order $\pi = (\pi_1, \dots, \pi_{|Pv|})$ on Pv and define the associative fold

$$\bar{\mu}_1(s) := pdf(s, \pi_1), \quad \bar{\mu}_{k+1}(s) := T_{pCF}(\bar{\mu}_k(s), pdf(s, \pi_{k+1}); \pi_k, \pi_{k+1}),$$

finally setting $\bar{\mu}(s) := \bar{\mu}_{|Pv|}(s)$.

The P-FITSM choice rule selects

$$s^* \in \arg \max_{s \in S} \bar{\mu}(s),$$

or, more broadly, uses $\bar{\mu}$ as an objective/score inside portfolio or optimization constraints.

Example 28 (PFITSM for managed-database selection). Consider three managed database services

$$S = \{\text{DB-Alpha}, \text{DB-Beta}, \text{DB-Gamma}\}.$$

We concentrate on the plithogenic evaluation layer from Definition 27; the FITSM components $(I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi)$ are assumed to be present but will not be used explicitly below.

Let the attribute be the *service evaluation facet* v with

$$Pv = \{Q, U, R, \text{Sec}, \text{Sus}\},$$

corresponding to Quality, Utility (economic benefit/cost), Reliability, Security, and Sustainability. From normalized KPIs we obtain facetwise memberships

$$\mu_Q, \mu_U, \mu_R, \mu_{\text{Sec}}, \mu_{\text{Sus}} : S \rightarrow [0, 1],$$

and we set $pdf(s, a)$ equal to the corresponding $\mu_a(s)$ (as permitted in Definition 27):

Service	μ_Q	μ_U	μ_R	μ_{Sec}	μ_{Sus}
DB-Alpha	0.82	0.65	0.97	0.90	0.55
DB-Beta	0.78	0.80	0.88	0.72	0.70
DB-Gamma	0.70	0.55	0.92	0.85	0.90

Next, choose a symmetric contradiction map $pCF : Pv \times Pv \rightarrow [0, 1]$ with diagonal zeros. The nontrivial entries used later (upper-triangular form) are listed in Table 2; symmetry supplies the remaining entries.

Fix the folding order

$$\pi = (R, \text{Sec}, Q, U, \text{Sus})$$

on Pv . Take the product t -norm $T(p, q) = pq$ and the probabilistic-sum t -conorm $S(p, q) = p + q - pq$. For adjacent facets (a, b) we use the standard plithogenic mixer

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q).$$

TABLE 2. Selected contradiction degrees $pCF(a, b)$ among facets.

$pCF(a, b)$	Q	U	R	Sec	Sus
Q	0	0.30	0.20	0.10	0.20
U		0	0.70	0.80	0.50
R			0	0.20	0.30
Sec				0	0.20
Sus					0

For each $s \in S$, define the fold by

$$\bar{\mu}_1(s) := pdf(s, \pi_1), \quad \bar{\mu}_{k+1}(s) := T_{pCF}(\bar{\mu}_k(s), pdf(s, \pi_{k+1}); \pi_k, \pi_{k+1}),$$

and finally $\bar{\mu}(s) := \bar{\mu}_{|P_v|}(s) \in [0, 1]$.

Step-by-step computation for DB-Alpha. Using $\pi = (R, Sec, Q, U, Sus)$ and the values above:

$$\begin{aligned}
\bar{\mu}_1 &= \mu_R = 0.97, \\
\bar{\mu}_2 &= T_{pCF}(0.97, 0.90; R, Sec) \\
&= (1 - 0.20)(0.97 \cdot 0.90) \\
&\quad + 0.20(0.97 + 0.90 - 0.97 \cdot 0.90) \\
&= 0.8 \cdot 0.873 + 0.2 \cdot 0.997 = 0.8978, \\
\bar{\mu}_3 &= T_{pCF}(0.8978, 0.82; Sec, Q) \\
&= (1 - 0.10)(0.8978 \cdot 0.82) \\
&\quad + 0.10(0.8978 + 0.82 - 0.8978 \cdot 0.82) \\
&= 0.9 \cdot 0.736196 + 0.1 \cdot 0.981604 = 0.7607368, \\
\bar{\mu}_4 &= T_{pCF}(0.7607368, 0.65; Q, U) \\
&= (1 - 0.30)(0.7607368 \cdot 0.65) \\
&\quad + 0.30(0.7607368 + 0.65 - 0.7607368 \cdot 0.65) \\
&= 0.7 \cdot 0.49447892 + 0.3 \cdot 0.91625788 = 0.621012608, \\
\bar{\mu}_5 &= T_{pCF}(0.621012608, 0.55; U, Sus) \\
&= (1 - 0.50)(0.621012608 \cdot 0.55) \\
&\quad + 0.50(0.621012608 + 0.55 - 0.621012608 \cdot 0.55) \\
&= 0.5 \cdot 0.3415569344 + 0.5 \cdot 0.8294556736 = 0.585506304.
\end{aligned}$$

Applying the same fold to DB-Beta and DB-Gamma gives

$$\bar{\mu}(\text{DB-Alpha}) = 0.5855, \quad \bar{\mu}(\text{DB-Beta}) = 0.6513, \quad \bar{\mu}(\text{DB-Gamma}) = 0.6920,$$

where we round to 4 significant digits for readability.² Hence the PFITSM selection is

$$s^* \in \arg \max_{s \in S} \bar{\mu}(s) = \{\text{DB-Gamma}\}.$$

Because pCF locally interpolates between conjunctive behavior (T) and disjunctive behavior (S), the fusion explicitly reflects facet conflicts, for example U versus Sec with $pCF(U, Sec) = 0.80$ and U versus R with $pCF(U, R) = 0.70$, while nearly compatible pairs such as Sec versus Q with 0.10 remain mostly conjunctive. This illustrates how PFITSM extends fuzzy ITSM scoring by attaching an explicit contradiction layer and a controlled plithogenic fusion mechanism.

Remark 29 (Well-posedness). *Because $T, S : [0, 1]^2 \rightarrow [0, 1]$ and $pCF(a, b) \in [0, 1]$, the mixture T_{pCF} maps $[0, 1]^2$ to $[0, 1]$. Consequently, each intermediate value $\bar{\mu}_k(s) \in [0, 1]$ and the final score $\bar{\mu} : S \rightarrow [0, 1]$ are*

²Exact values: 0.585506304, 0.6513239104, and 0.69196112, respectively.

well-defined. Moreover, if T and S are nondecreasing in each argument (as for standard t -(co)norms), then T_{pCF} and the resulting fold are nondecreasing in every input membership $\text{pdf}(s, a)$.

Fix an associative, nondecreasing fusion map $A_{\text{FITSM}} : [0, 1]^m \rightarrow [0, 1]$ used by the baseline FITSM to combine normalized facet scores (e.g. a product, a minimum, or a weighted Archimedean construction). Define

$$\mu_{\text{FITSM}}(s) := A_{\text{FITSM}}(\mu_Q(s), \mu_U(s), \mu_R(s), \dots) \in [0, 1].$$

Theorem 30 (Reduction to FITSM). *Assume PFITSM satisfies:*

- (1) $pCF \equiv 0$ on Pv ;
- (2) the chosen t -norm T and the fixed order π are such that the fold reproduces A_{FITSM} on the ordered list $\{\mu_{\bullet}(s)\}_{a \in Pv}$ (for example, $T = \min$ for a conjunctive FITSM, or $T(p, q) = pq$ for a multiplicative FITSM);
- (3) S is arbitrary (it is not invoked when $pCF \equiv 0$).

Then for all $s \in S$, one has $\bar{\mu}(s) = \mu_{\text{FITSM}}(s)$. In particular,

$$\arg \max_{s \in S} \bar{\mu}(s) = \arg \max_{s \in S} \mu_{\text{FITSM}}(s).$$

Proof: When $pCF \equiv 0$, the mixer reduces to $T_{pCF}(p, q; a, b) = T(p, q)$ for every pair (a, b) . Therefore the plithogenic fold becomes a pure T -fold along the fixed order π . By the assumption on T and π , this fold coincides with A_{FITSM} applied to $(\mu_Q(s), \mu_U(s), \mu_R(s), \dots)$. Hence $\bar{\mu}(s) = \mu_{\text{FITSM}}(s)$ for all $s \in S$, and the maximizers agree. $\square$

Theorem 31 (Plithogenic fuzzy set structure of P-FITSM). *Let PFITSM be as in Definition 27. Put $P := S$ and keep the same (v, Pv, pdf, pCF) . Then*

$$PS := (P, v, Pv, \text{pdf}, pCF)$$

is a plithogenic fuzzy set.

Proof: By construction, $\text{pdf} : S \times Pv \rightarrow [0, 1]$ is well-defined, and $pCF : Pv \times Pv \rightarrow [0, 1]$ is symmetric with $pCF(a, a) = 0$ for all $a \in Pv$. These are exactly the defining axioms for a plithogenic fuzzy set in the scalar appurtenance/contradiction setting. Therefore PS is a plithogenic fuzzy set. $\square$

2.4 Upside-Down Logic in Plithogenic Fuzzy IT Service Management

This subsection introduces a rigorous *Upside-Down* operator for the Plithogenic Fuzzy IT Service Management (PFITSM) setting. The operator is triggered by an *anchor facet* and a *contradiction threshold*: facets that strongly contradict the anchor have their acceptability complemented, and the corresponding anchor-facet contradictions are reset. We then verify that, under a natural mid-cut truth valuation on elementary acceptance statements, the operator enforces the characteristic truth $\leftrightarrow$ falsity inversion of Upside-Down Logic on the activated facets.

Notation 32 (PFITSM data and facetwise memberships). Let

$$\text{PFITSM} = (S, v, Pv, \text{pdf}, pCF, \text{Agg}, \dots)$$

be a PFITSM instance, where S is a finite set of services, v is the fixed attribute (“service facet”), Pv is its value domain (e.g., availability, cost, customer satisfaction, SLA compliance), $\text{pdf} : S \times Pv \rightarrow [0, 1]$ is the facetwise membership (fuzzy acceptance), and $pCF : Pv \times Pv \rightarrow [0, 1]$ is a symmetric contradiction map with $pCF(a, a) = 0$ for every $a \in Pv$. The aggregator Agg combines $\{\text{pdf}(s, a)\}_{a \in Pv}$ into a global membership $\bar{\mu}(s) \in [0, 1]$.

We write

$$\mu(s \mid a) := \text{pdf}(s, a) \in [0, 1], \quad c(a, b) := pCF(a, b) \in [0, 1].$$

When it is convenient to be explicit, we assume Agg is built from a t -norm T and a t -conorm S through the contradiction-modulated mixer

$$T_c(p, q; a, b) := (1 - c(a, b)) T(p, q) + c(a, b) S(p, q),$$

and then folded associatively across the facets in Pv .

Definition 33 (Anchor-threshold Upside-Down operator on PFITSM). Fix an *anchor facet* $b \in Pv$ and a *contradiction threshold* $\tau \in [0, 1]$. Define the activation locus

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The *Upside-Down transform* $U_{b,\tau}$ maps PFITSM to a new PFITSM instance

$$\text{PFITSM}^{U_{b,\tau}} = (S, v, Pv, pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}}, \text{Agg}^{U_{b,\tau}}, \dots),$$

where, for every service $s \in S$ and facet $a \in Pv$,

$$pdf^{U_{b,\tau}}(s, a) := \begin{cases} 1 - \mu(s | a), & a \in A_\tau(b), \\ \mu(s | a), & a \notin A_\tau(b), \end{cases}$$

$$pCF^{U_{b,\tau}}(u, v) := \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

Finally, $\text{Agg}^{U_{b,\tau}}$ is obtained from Agg by substituting $pdf^{U_{b,\tau}}$ and $pCF^{U_{b,\tau}}$ (the underlying t -(co)norms and the folding policy are unchanged).

Example 34 (Cost-anchored FinOps pivot for a cloud email service). Service and facets. Let $S = \{s_{\text{email}}\}$ be a managed cloud email service and consider

$$Pv = \{\text{Ava}, \text{Sec}, \text{Lat}, \text{CS}, \text{Cost}\},$$

representing Availability, Security compliance, Latency, Customer Satisfaction, and Budget Adherence (Cost). Choose the anchor $b = \text{Cost}$ and set $\tau = 0.7$.

Contradictions relative to the anchor. Assume (with symmetric extension)

$$c(\text{Ava}, \text{Cost}) = 0.85, \quad c(\text{Sec}, \text{Cost}) = 0.75, \quad c(\text{Lat}, \text{Cost}) = 0.60, \quad c(\text{CS}, \text{Cost}) = 0.50, \quad c(\text{Cost}, \text{Cost}) = 0.$$

Hence

$$A_\tau(b) = \{\text{Ava}, \text{Sec}\}.$$

Baseline facetwise acceptances. Suppose

$$\mu(s_{\text{email}} | \text{Ava}) = 0.97, \quad \mu(s_{\text{email}} | \text{Sec}) = 0.92, \quad \mu(s_{\text{email}} | \text{Lat}) = 0.88,$$

$$\mu(s_{\text{email}} | \text{CS}) = 0.90, \quad \mu(s_{\text{email}} | \text{Cost}) = 0.35.$$

Apply $U_{b,\tau}$. By Definition 33, the activated facets are complemented and the corresponding contradictions to the anchor are reset:

$$pdf^{U_{b,\tau}}(s_{\text{email}}, a) = \begin{cases} 1 - \mu(s_{\text{email}} | a), & a \in \{\text{Ava}, \text{Sec}\}, \\ \mu(s_{\text{email}} | a), & a \in \{\text{Lat}, \text{CS}, \text{Cost}\}, \end{cases}$$

$$pCF^{U_{b,\tau}}(\text{Ava}, \text{Cost}) = 0, \quad pCF^{U_{b,\tau}}(\text{Sec}, \text{Cost}) = 0,$$

and $pCF^{U_{b,\tau}}(u, v) = pCF(u, v)$ for all other pairs.

Numerical values after the transform.

$$\begin{array}{ll} \text{Ava} & : 1 - 0.97 = 0.03, \\ \text{Sec} & : 1 - 0.92 = 0.08, \\ \text{Lat} & : 0.88 \text{ (unchanged)}, \\ \text{CS} & : 0.90 \text{ (unchanged)}, \\ \text{Cost} & : 0.35 \text{ (unchanged)}. \end{array}$$

Interpretation. Under a FinOps review that anchors on *Budget Adherence*, the highly cost-contradictory facets (*Availability* and *Security*) undergo acceptance inversion: large baseline memberships become small post-transform memberships. The reset in $pCF^{U_{b,\tau}}$ prevents immediate reactivation at the same threshold in subsequent analyses (e.g., repeated aggregation).

Example 35 (Privacy-first shift for an AI chatbot platform). Service and facets. Let $S = \{s_{\text{chatbot}}\}$ be an AI chatbot service and consider

$$Pv = \{\text{GDPR, Pers, Telem, Perf, DevVel}\},$$

standing for GDPR Compliance, Personalization, Telemetry/Logging, Performance, and Developer Velocity. Choose the privacy anchor $b = \text{GDPR}$ and set $\tau = 0.8$.

Contradictions relative to the anchor. Assume (with symmetric extension)

$$\begin{aligned} c(\text{Pers, GDPR}) &= 0.90, & c(\text{Telem, GDPR}) &= 0.85, \\ c(\text{Perf, GDPR}) &= 0.40, & c(\text{DevVel, GDPR}) &= 0.30, \\ c(\text{GDPR, GDPR}) &= 0. \end{aligned}$$

Then

$$A_\tau(b) = \{\text{Pers, Telem}\}.$$

Baseline facetwise acceptances. Suppose

$$\begin{aligned} \mu(s_{\text{chatbot}} \mid \text{Pers}) &= 0.86, & \mu(s_{\text{chatbot}} \mid \text{Telem}) &= 0.80, \\ \mu(s_{\text{chatbot}} \mid \text{Perf}) &= 0.82, & \mu(s_{\text{chatbot}} \mid \text{DevVel}) &= 0.75, \\ \mu(s_{\text{chatbot}} \mid \text{GDPR}) &= 0.45. \end{aligned}$$

Apply $U_{b,\tau}$. The activated facets are complemented and their contradictions to the anchor are reset:

$$pdf^{U_{b,\tau}}(s_{\text{chatbot}}, a) = \begin{cases} 1 - \mu(s_{\text{chatbot}} \mid a), & a \in \{\text{Pers, Telem}\}, \\ \mu(s_{\text{chatbot}} \mid a), & a \in \{\text{Perf, DevVel, GDPR}\}, \end{cases}$$

$$pCF^{U_{b,\tau}}(\text{Pers, GDPR}) = 0, \quad pCF^{U_{b,\tau}}(\text{Telem, GDPR}) = 0,$$

and $pCF^{U_{b,\tau}}(u, v) = pCF(u, v)$ otherwise.

Numerical values after the transform.

$$\begin{aligned} \text{Pers} &: 1 - 0.86 = 0.14, \\ \text{Telem} &: 1 - 0.80 = 0.20, \\ \text{Perf} &: 0.82 \text{ (unchanged)}, \\ \text{DevVel} &: 0.75 \text{ (unchanged)}, \\ \text{GDPR} &: 0.45 \text{ (unchanged)}. \end{aligned}$$

Interpretation. With GDPR as the anchor and a strict threshold, *Personalization* and *Telemetry* are treated as high-conflict facets and thus invert under $U_{b,\tau}$: what was previously advantageous becomes disadvantageous relative to compliance. The contradiction reset yields a stabilized post-transform context for further evaluation (e.g., refolding or constrained optimization).

Lemma 36 (Well-posedness). *The transformed instance $\text{PFITSM}^{U_{b,\tau}}$ is well-defined: for all $(s, a) \in S \times Pv$, one has $pdf^{U_{b,\tau}}(s, a) \in [0, 1]$; the updated map $pCF^{U_{b,\tau}}$ remains symmetric and satisfies $pCF^{U_{b,\tau}}(a, a) = 0$; in particular, the resulting aggregated membership $\bar{\mu}^{U_{b,\tau}} : S \rightarrow [0, 1]$ is defined.*

Proof: Because $\mu(s \mid a) \in [0, 1]$, its complement $1 - \mu(s \mid a)$ also lies in $[0, 1]$, so $pdf^{U_{b,\tau}}(s, a) \in [0, 1]$ for every (s, a) . The definition of $pCF^{U_{b,\tau}}$ only changes selected off-diagonal entries by setting them to 0, hence symmetry and diagonal zeros are preserved. If **Agg** is constructed from standard t -(co)norm mixers, each binary step maps $[0, 1]^2$ to $[0, 1]$, and so the associative fold produces values in $[0, 1]$ as well. $\square$

Notation 37 (Mid-cut language and valuation). Let $\mathcal{L}$ consist of *elementary acceptance statements*

$$\text{Acc}_{1/2}(s, a) \quad \text{“service } s \text{ is acceptable on facet } a \text{ at the mid-cut”}.$$

For a PFITSM instance, define the mid-cut valuation

$$v_{b,\tau}(\text{Acc}_{1/2}(s, a)) := \begin{cases} \text{True}, & \mu(s \mid a) \geq \frac{1}{2}, \\ \text{False}, & \mu(s \mid a) < \frac{1}{2}. \end{cases}$$

After applying $U_{b,\tau}$, we evaluate the same predicate using $\text{pdf}^{U_{b,\tau}}$, thereby obtaining the post-transform valuation $v'_{b,\tau}$.

Theorem 38 (Truth–falsity inversion on activated facets). *Fix (b, τ) and let $s \in S$, $a \in Pv$ satisfy $\mu(s \mid a) \neq \frac{1}{2}$. Then*

$$a \in A_\tau(b) \implies v'_{b,\tau}(\text{Acc}_{1/2}(s, a)) = \neg v_{b,\tau}(\text{Acc}_{1/2}(s, a)),$$

and

$$a \notin A_\tau(b) \implies v'_{b,\tau}(\text{Acc}_{1/2}(s, a)) = v_{b,\tau}(\text{Acc}_{1/2}(s, a)).$$

Proof: If $a \notin A_\tau(b)$, then by definition $\text{pdf}^{U_{b,\tau}}(s, a) = \mu(s \mid a)$, so the truth value under the mid-cut predicate is unchanged.

If $a \in A_\tau(b)$, then $\text{pdf}^{U_{b,\tau}}(s, a) = 1 - \mu(s \mid a)$. Whenever $\mu(s \mid a) \neq \frac{1}{2}$, the numbers $\mu(s \mid a)$ and $1 - \mu(s \mid a)$ lie on opposite sides of $\frac{1}{2}$, hence the mid-cut truth value is inverted: True becomes False and False becomes True. $\square$

3 Additional Results: U -Model for Risk Management and IT Service Management

As an additional contribution we introduce a U -model for Risk Management and IT Service Management. This is an extension based on the Uncertain Set template [30, 31].

An *Uncertain Set* (briefly, a U -Set) assigns to each element of a universe an uncertainty value taken from a prescribed *degree-domain*. By varying the degree-domain one can regard fuzzy, intuitionistic fuzzy, neutrosophic, plithogenic, and related degree-based schemes as instances of a single pattern [31].

Definition 39 (Uncertain Set (U-Set)). [31] Let X be a nonempty set and let M be an uncertainty model with degree-domain

$$\text{Dom}(M) \subseteq [0, 1]^k$$

for some integer $k \geq 1$. A U -Set of type M on X is a mapping

$$\mu_M : X \longrightarrow \text{Dom}(M).$$

For each $x \in X$, the element $\mu_M(x) \in \text{Dom}(M)$ is called the M -membership degree (or M -uncertainty value) of x . With suitable choices of M , this definition recovers standard degree-based set models (fuzzy, intuitionistic fuzzy, neutrosophic, plithogenic, and others).

3.1 Uncertain Risk Management

Uncertain Risk Management evaluates decisions by mapping each option to an uncertainty membership over a degree-domain, generalizing fuzzy risk measures. We now formulate Risk Management directly as a U -Set on the decision domain.

Definition 40 (Uncertain Risk Management (URM)). Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $D \subseteq \mathbb{R}^n$ be a nonempty decision set. Let

$$L : D \longrightarrow L^\infty(\Omega, \mathcal{F}, \mathbb{P}), \quad x \longmapsto L(x),$$

assign to each decision $x \in D$ an essentially bounded loss random variable, and let

$$\rho : L^\infty(\Omega, \mathcal{F}, \mathbb{P}) \longrightarrow \mathbb{R}$$

be a (coherent) risk measure in the usual crisp sense.

Fix an uncertainty model M with degree-domain $\text{Dom}(M) \subseteq [0, 1]^k$. Let

$$\Phi_M : \mathbb{R} \longrightarrow \text{Dom}(M)$$

be a continuous mapping which is *strictly decreasing in risk* in the coordinatewise order on $[0, 1]^k$, i.e., for $r_1 < r_2$ we have $\Phi_M(r_2) \prec \Phi_M(r_1)$ (strictly smaller in at least one coordinate and no larger in any coordinate).

Define the *URM acceptance map* by

$$\mu_M(x) := \Phi_M(\rho(L(x))) \in \text{Dom}(M), \quad x \in D.$$

Thus (D, μ_M) is the *uncertain decision set* of type M .

To compare uncertainty values in $\text{Dom}(M)$, fix a ranking functional

$$\text{Score}_M : \text{Dom}(M) \longrightarrow \mathbb{R}$$

that is nondecreasing in each coordinate. The *URM decision problem* is

$$x^* \in \arg \max_{x \in D} \text{Score}_M(\mu_M(x)).$$

Proposition 41 (Well-definedness and U -Set structure of URM). *In Definition 40, the mapping $\mu_M : D \rightarrow \text{Dom}(M)$ is well-defined. Hence (D, μ_M) is a U -Set of type M on the universe D .*

Proof: For any $x \in D$, $L(x) \in L^\infty(\Omega, \mathcal{F}, \mathbb{P})$ by assumption, so $\rho(L(x)) \in \mathbb{R}$. Since $\Phi_M : \mathbb{R} \rightarrow \text{Dom}(M)$, we have $\mu_M(x) = \Phi_M(\rho(L(x))) \in \text{Dom}(M)$. Therefore μ_M is a mapping from D to $\text{Dom}(M)$, i.e., a U -Set of type M on D . $\square$

Theorem 42 (URM generalizes Fuzzy Risk Management). *Assume $k = 1$ and $\text{Dom}(M) = [0, 1]$. Suppose $\Phi_M = \Phi$ is a continuous strictly decreasing mapping $\Phi : \mathbb{R} \rightarrow [0, 1]$, and choose $\text{Score}_M(t) = t$ for $t \in [0, 1]$. Then the URM acceptance map becomes*

$$\mu_M(x) = \Phi(\rho(L(x))) \in [0, 1],$$

and the URM optimization

$$\arg \max_{x \in D} \text{Score}_M(\mu_M(x))$$

coincides with the fuzzy risk management rule

$$\arg \max_{x \in D} \Phi(\rho(L(x)))$$

(as in the fuzzy framework).

Proof: Under $k = 1$ and $\text{Dom}(M) = [0, 1]$, the map Φ_M is a scalar map $\Phi : \mathbb{R} \rightarrow [0, 1]$. Hence Definition 40 gives

$$\mu_M(x) = \Phi(\rho(L(x))) \in [0, 1].$$

With $\text{Score}_M(t) = t$, maximizing $\text{Score}_M(\mu_M(x))$ is exactly maximizing $\mu_M(x)$, so

$$\arg \max_{x \in D} \text{Score}_M(\mu_M(x)) = \arg \max_{x \in D} \mu_M(x) = \arg \max_{x \in D} \Phi(\rho(L(x))).$$

This is precisely the fuzzy risk management objective. $\square$

3.2 Uncertain IT Service Management

Uncertain IT Service Management models service evaluation by assigning each service a multi-dimensional uncertainty membership from a chosen degree-domain. We now introduce an uncertain-set extension of IT Service Management, in parallel to the URM construction.

Definition 43 (Uncertain IT Service Management (UITSM)). Let S be a finite set of IT services, I a finite set of infrastructure components, and P a finite set of ITSM processes. Let

$$M : S \rightarrow 2^I \times 2^P, \quad s \mapsto (I_s, P_s),$$

be an allocation mapping, let C denote the relevant cost/budget parameters, let

$$\varphi : 2^I \rightarrow [0, 1]$$

An International Journal of Computational Intelligence Methods, and Applications

be an aggregation function for component-level reliability/availability, and let

$$\psi : P \times S \rightarrow \{0, 1\}$$

be a process compliance predicate.

Fix $m \geq 1$ normalized evaluation facets (KPIs). Concretely, let

$$\mu_1, \dots, \mu_m : S \rightarrow [0, 1]$$

be facetwise acceptance scores for each service (examples: normalized quality, utility, reliability, security, compliance, sustainability, customer satisfaction, latency, etc.). Write

$$\mu(s) := (\mu_1(s), \dots, \mu_m(s)) \in [0, 1]^m.$$

Fix an uncertainty model M with degree-domain $\text{Dom}(M) \subseteq [0, 1]^k$. Let

$$\Phi_M : [0, 1]^m \rightarrow \text{Dom}(M)$$

be a continuous fusion map producing an M -uncertainty value from the facet vector. Define the *UITSM membership map* by

$$\mu_M(s) := \Phi_M(\mu(s)) \in \text{Dom}(M), \quad s \in S.$$

Thus (S, μ_M) is a U -Set of type M on the universe of services.

To select services, fix a ranking functional

$$\text{Score}_M : \text{Dom}(M) \rightarrow \mathbb{R}$$

that is nondecreasing in each coordinate, and define the *UITSM decision rule* by

$$s^* \in \arg \max_{s \in S} \text{Score}_M(\mu_M(s)).$$

Proposition 44 (Well-definedness and U -Set structure of UITSM). *In Definition 43, the mapping $\mu_M : S \rightarrow \text{Dom}(M)$ is well-defined. Hence (S, μ_M) is a U -Set of type M on the universe S .*

Proof: For each $s \in S$, we have $\mu(s) \in [0, 1]^m$ by construction of the normalized facet scores. Since $\Phi_M : [0, 1]^m \rightarrow \text{Dom}(M)$, it follows that $\mu_M(s) = \Phi_M(\mu(s)) \in \text{Dom}(M)$. Therefore μ_M is a mapping from S to $\text{Dom}(M)$, i.e., a U -Set of type M on S . $\square$

Theorem 45 (UITSM generalizes Fuzzy IT Service Management). *Assume $k = 1$ and $\text{Dom}(M) = [0, 1]$. Let $A_{\text{FITSM}} : [0, 1]^m \rightarrow [0, 1]$ be any associative, nondecreasing fusion used in the fuzzy ITSM layer to combine the normalized facet scores into a single fuzzy acceptance value, and define*

$$\mu_{\text{FITSM}}(s) := A_{\text{FITSM}}(\mu(s)) \in [0, 1].$$

Choose $\Phi_M = A_{\text{FITSM}}$ and $\text{Score}_M(t) = t$ on $[0, 1]$. Then for all $s \in S$,

$$\mu_M(s) = \mu_{\text{FITSM}}(s),$$

and the UITSM selection rule

$$\arg \max_{s \in S} \text{Score}_M(\mu_M(s))$$

coincides with the fuzzy ITSM rule $\arg \max_{s \in S} \mu_{\text{FITSM}}(s)$.

Proof: Under $k = 1$ and $\text{Dom}(M) = [0, 1]$, the map Φ_M is scalar-valued. With $\Phi_M = A_{\text{FITSM}}$, Definition 43 gives

$$\mu_M(s) = \Phi_M(\mu(s)) = A_{\text{FITSM}}(\mu(s)) = \mu_{\text{FITSM}}(s) \in [0, 1].$$

With $\text{Score}_M(t) = t$, maximizing $\text{Score}_M(\mu_M(s))$ is the same as maximizing $\mu_M(s)$, hence

$$\arg \max_{s \in S} \text{Score}_M(\mu_M(s)) = \arg \max_{s \in S} \mu_M(s) = \arg \max_{s \in S} \mu_{\text{FITSM}}(s).$$

Therefore the UITSM rule reduces to the fuzzy ITSM rule in this specialization. $\square$

Remark 46 (Where the uncertain-set idea enters). *In both URM and UITSM the central object is a membership mapping into a degree-domain:*

$$\mu_M : D \rightarrow \text{Dom}(M) \quad (\text{URM}), \quad \mu_M : S \rightarrow \text{Dom}(M) \quad (\text{UITSM}).$$

Allowing $\text{Dom}(M) \subseteq [0, 1]^k$ with $k > 1$ permits multi-component uncertainty values (e.g., pairs/triples recording acceptance, hesitation, indeterminacy, or other model-dependent coordinates). Theorems 42 and 45 show that the fuzzy models are recovered as the special case $k = 1$, $\text{Dom}(M) = [0, 1]$.

4 Conclusion

In this paper, we formally introduced *Plithogenic Fuzzy Risk Management* and *Plithogenic Fuzzy IT Service Management* and examined their core structural properties. In addition, we proposed a *U*-model for both Risk Management and IT Service Management as a further conceptual extension. For reference, Table 3 summarizes the comparison between Fuzzy Risk Management (FRM) and Plithogenic Fuzzy Risk Management (PFRM), while Table 4 presents an overview of the comparison between Fuzzy IT Service Management (FITSM) and Plithogenic Fuzzy IT Service Management (PFITSM).

TABLE 3. Concise comparison of Fuzzy Risk Management (FRM) and Plithogenic Fuzzy Risk Management (PFRM).

Aspect	Fuzzy Risk Management (FRM)	Plithogenic Fuzzy Risk Management (PFRM)
Decision universe	Decisions $x \in D \subseteq \mathbb{R}^n$.	Decisions $x \in D \subseteq \mathbb{R}^n$ (same universe).
Evaluation granularity	Single acceptance degree $\mu_{\mathcal{A}}(x) \in [0, 1]$ for each decision.	Facetwise acceptance degrees $pdf(x, a) \in [0, 1]$ for each facet $a \in Pv$, then aggregated to $\bar{\mu}(x) \in [0, 1]$.
Facet/model structure	Implicitly single criterion: one loss map L , one risk measure ρ , one satisfaction map Φ .	Explicit multi-facet modeling: for each $a \in Pv$, (L_a, ρ_a, Φ_a) defines $pdf(x, a) = \Phi_a(\rho_a(L_a(x)))$.
Uncertainty representation	Fuzziness via $\Phi(\rho(L(x)))$ (gradual acceptability).	Fuzziness per facet plus <i>inter-facet contradiction</i> through $pCF : Pv \times Pv \rightarrow [0, 1]$.
Contradiction/conflict handling	No explicit conflict parameter among criteria.	Explicit contradiction map $pCF(a, b)$ (symmetric, $pCF(a, a) = 0$) quantifies conflict between facets.
Aggregation mechanism	No separate aggregation (single membership).	Plithogenic aggregation Agg mixes a t -norm T and t -conorm S using $pCF(a, b)$, producing $\bar{\mu}(x)$.
Optimization objective	$x^* \in \arg \max_{x \in D} \mu_{\mathcal{A}}(x)$ (equivalently minimize $\rho(L(x))$ under monotone Φ).	$x^* \in \arg \max_{x \in D} \bar{\mu}(x)$ (overall acceptability after contradiction-modulated fusion).
Reduction condition	Baseline model.	Reduces to FRM when $Pv = \{a_0\}$ and $pCF \equiv 0$, giving $\bar{\mu}(x) = pdf(x, a_0)$.

Future directions include developing extended models based on Graphs [32], HyperGraphs [33], and SuperHyperGraphs [34, 35]. Since this work is entirely theoretical, we also hope that subsequent studies will undertake computational experiments, machine-learning approaches, and quantitative analyses carried out by domain specialists.

Declarations

Ethics Approval and Consent to Participate

The results, data, and figures presented in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and no permissions are required.

TABLE 4. Concise comparison of Fuzzy IT Service Management (FITSM) and Plithogenic Fuzzy IT Service Management (PFITSM).

Aspect	Fuzzy IT Service Management (FITSM)	Plithogenic Fuzzy IT Service Management (P-FITSM / PFITSM)
Service universe	Services $s \in S$.	Services $s \in S$ (same universe).
Core ITSM structure	Uses $(S, I, P, M, C, \varphi, \psi)$ plus fuzzy evaluation functions (e.g. $\tilde{Q}, \tilde{U}, \tilde{R}$).	Keeps the same ITSM backbone and adds a plithogenic layer $(v, Pv, pdf, pCF, \text{Agg})$.
Facet/model structure	Typically evaluates a service by a set of fuzzy facet scores (e.g. Quality/Utility/Reliability), then combines them by a chosen fusion rule.	Makes facets explicit as attribute values $a \in Pv$ (canonical $Pv = \{Q, U, R\}$, extensible), with facetwise memberships $pdf(s, a) \in [0, 1]$.
Uncertainty representation	Fuzziness mainly comes from fuzzy-valued assessments (e.g. $\tilde{Q}(s)$) mapped to normalized scores.	Fuzziness per facet, plus explicit <i>contradiction among facets</i> via $pCF(a, b) \in [0, 1]$.
Contradiction/conflict handling	No explicit contradiction parameter between facets (trade-offs handled implicitly by the fusion choice).	Explicit contradiction function pCF (symmetric, diagonal zero) controls how strongly conflicts affect fusion facet-by-facet.
Aggregation mechanism	A baseline associative fusion $A_{\text{FITSM}} : [0, 1]^m \rightarrow [0, 1]$ (e.g. product, min, weighted norm) yields an overall membership $\mu_{\text{FITSM}}(s)$.	Plithogenic aggregator Agg folds facet scores using $T_{pCF} = (1 - pCF)T + pCFS$ to produce $\bar{\mu}(s)$.
Decision rule	$s^* \in \arg \max_{s \in S} \mu_{\text{FITSM}}(s)$.	$s^* \in \arg \max_{s \in S} \bar{\mu}(s)$ (or used inside constrained portfolio/optimization).
Reduction condition	Baseline model.	Reduces to FITSM when $pCF \equiv 0$ and T is chosen so that the fold coincides with A_{FITSM} , giving $\bar{\mu}(s) = \mu_{\text{FITSM}}(s)$.

Consent for Publication

This article does not contain any studies involving human participants or animals performed by any of the authors.

Availability of Data and Materials

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests in this research.

Funding

This research was not supported by any funding agency or institute.

Acknowledgment

The author expresses gratitude to the editorial team and reviewers, as well as the corresponding author, for their valuable advice, assessment, and review during the study period.

References

- [1] Antonios Paraskevas, Michael Madas, Florentin Smarandache, and Takaaki Fujita. Utility-based upside-down logic: A neutrosophic decision-making framework under uncertainty. *Journal of Decisions and Operations Research*, 2025.

- [2] Takaaki Fujita and Florentin Smarandache. Introduction to upside-down logic: Its deep relation to neutrosophic logic and applications. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond (Third Volume)*, 2024.
- [3] Florentin Smarandache. *Upside-Down Logics: Falsification of the Truth and Truthification of the False*. Infinite Study, 2024.
- [4] Takaaki Fujita and Arif Mehmood. A study on the effectiveness of contradiction values in upside-down logic and deplithogenication within plithogenic sets. *Neutrosophic Computing and Machine Learning*, 40(1):28–65, 2025.
- [5] Richard Kaye. *Models of Peano Arithmetic*. Clarendon Press, Oxford, 1991.
- [6] Fazeelat Sultana, Muhammad Gulistan, Mumtaz Ali, Naveed Yaqoob, Muhammad Khan, Tabasam Rashid, and Tauseef Ahmed. A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, 14(10):13139–13159, 2023.
- [7] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.
- [8] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.
- [9] Krassimir T Atanassov. Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, 39(5):5981–5986, 2020.
- [10] Vicenç Torra and Yasuo Narukawa. On hesitant fuzzy sets and decision. In *2009 IEEE international conference on fuzzy systems*, pages 1378–1382. IEEE, 2009.
- [11] Haibin Wang, Florentin Smarandache, Yanqing Zhang, and Rajshekhar Sunderraman. *Single valued neutrosophic sets*. Infinite study, 2010.
- [12] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Single valued neutrosophic graphs. *Journal of New theory*, (10):86–101, 2016.
- [13] Florentin Smarandache. *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study, 2018.
- [14] Raúl González Salas, Jhael Mikaela Gómez Ashqui, and Marco Paúl Medina Valencia. Assessing viral susceptibility in tilapia species using plithogenic fuzzy soft set: a multi-expert approach for tilv. *Neutrosophic Sets and Systems*, 89:185–196, 2025.
- [15] Nivetha Martin, Seyyed Ahmad Edalpathnath, Florentin Smarandache, et al. Extended plithogenic hypersoft sets in decision making on electric vehicle charging infrastructure. *Journal of Fuzzy Extension and Applications*, page e226517, 2025.
- [16] Takaaki Fujita, Raed Hatamleh, and Ahmed Salem Heilat. Contrasofter set and contrarough set with using upside-down logic. *Statistics, Optimization & Information Computing*, 14(6):3589–3632, 2025.
- [17] Michael Power. The risk management of everything. *The Journal of Risk Finance*, 5(3):58–65, 2004.
- [18] René M Stulz. Rethinking risk management. In *Corporate risk management*, pages 87–120. Columbia University Press, 2008.
- [19] Farnad Nasirzadeh, Abbas Afshar, Mostafa Khanzadi, and Susan Howick. Integrating system dynamics and fuzzy logic modelling for construction risk management. *Construction management and economics*, 26(11):1197–1212, 2008.
- [20] P Zlateva, D Velez, and L Raeva. A fuzzy logic method for assessment of risk management capability. *International Journal of Innovation, Management and Technology*, 6(4):260, 2015.
- [21] Takaaki Fujita, Arif Mehmood, and Arkan A Ghaib. A reconsideration note of the mathematical frameworks for fuzzy and neutrosophic risk management. *Management Science Advances*, 2(1):223–238, 2025.
- [22] Takaaki Fujita. A proposed mathematical framework for fuzzy it service management (F-ITSM) and neutrosophic it service management (N-ITSM). *Neutrosophic Systems with Applications*, 25(6), 2025. Article 2.
- [23] Jon Iden and Tom Roar Eikebrokk. Implementing it service management: A systematic literature review. *International journal of information management*, 33(3):512–523, 2013.
- [24] Rob Addy. *Effective IT service management: to ITIL and beyond!*, volume 29. Springer, 2007.
- [25] BC Potgieter, JH Botha, and C Lew. Evidence that use of the itil framework is effective. In *18th Annual conference of the national advisory committee on computing qualifications, Tauranga, NZ*. Citeseer, 2005.
- [26] Norita Ahmad and Zulkifli M Shamsudin. Systematic approach to successful implementation of itil. *Procedia computer science*, 17:237–244, 2013.
- [27] Dave Armes, Niklas Engelhart, Peter McKenzie, and Peter Wiggers. *SIAM: Principles and practices for service integration and management*. Van Haren, 2015.
- [28] Otso Virri. *Service Integration and Management (SIAM) Practices and Four Case Studies of Finnish Companies*. Metropolia Ammattikorkeakoulu, 2017.
- [29] Takaaki Fujita. A proposed framework for fuzzy and neutrosophic extensions of service integration and management (siam) and itil-based models. *Journal of Intelligent Decision Making and Granular Computing*, 1(1):161–198, 2025.
- [30] Takaaki Fujita and Florentin Smarandache. *A Dynamic Survey of Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Plithogenic, and Extensional Sets*. Neutrosophic Science International Association (NSIA), 2025.
- [31] Takaaki Fujita and Florentin Smarandache. A unified framework for u -structures and functorial structure: Managing super, hyper, superhyper, tree, and forest uncertain over/under/off models. *Neutrosophic Sets and Systems*, 91:337–380, 2025.
- [32] Reinhard Diestel. *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.
- [33] Yue Gao, Zizhao Zhang, Haojie Lin, Xibin Zhao, Shaoyi Du, and Changqing Zou. Hypergraph learning: Methods and practices. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(5):2548–2566, 2020.
- [34] Salomón Marcos Berrocal Villegas, Willner Montalvo Fritas, Carmen Rosa Berrocal Villegas, María Yissel Flores Fuentes Rivera, Roberto Espejo Rivera, Laura Daysi Bautista Puma, and Dante Manuel Macazana Fernández. Using plithogenic n -superhypergraphs to assess the degree of relationship between information skills and digital competencies. *Neutrosophic Sets and Systems*, 84(1):41, 2025.
- [35] Ehab Roshdy, Marwa Khashaba, and Mariam Emad Ahmed Ali. Neutrosophic super-hypergraph fusion for proactive cyberattack countermeasures: A soft computing framework. *Neutrosophic Sets and Systems*, 94:232–252, 2025.

Received: 03 Jun 2025, **Revised:** 23 Oct 2025,
Accepted: 19 Dec 2025, **Available online:** 21 Dec 2025.



© 2025 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).

Disclaimer/Publisher's Note: The perspectives, opinions, and data shared in all publications are the sole responsibility of the individual authors and contributors, and do not necessarily reflect the views of Sciences Force or the editorial team. Sciences Force and the editorial team disclaim any liability for potential harm to individuals or property resulting from the ideas, methods, instructions, or products referenced in the content.