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A Note on the Use of Plithogenic and Hyperplithogenic Sets for Weather Prediction

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Abstract

Many well-known formalisms for handling uncertainty—such as *Fuzzy Sets*, *Intuitionistic Fuzzy Sets*, *Neutrosophic Sets*, and *Plithogenic Sets*—have been widely investigated and employed in diverse application areas. Building on Plithogenic Sets, two higher-order extensions have been proposed: the *HyperPlithogenic Set*, which generalises the original Plithogenic structure, and the *SuperHyperPlithogenic Set*, which in turn subsumes both Plithogenic and HyperPlithogenic Sets.

This paper demonstrates how these three families—Plithogenic, HyperPlithogenic, and SuperHyperPlithogenic Sets—can be employed for weather forecasting. Because meteorological prediction has already been modelled within Fuzzy and Neutrosophic frameworks, extending the analysis to the richer Plithogenic hierarchy constitutes a natural and promising next step.

Keywords: Set Theory, Plithogenic Set, HyperPlithogenic Set, SuperhyperPlithogenic Set, Fuzzy Set

1 | Preliminaries

This section reviews the basic notions and terminology that will be used throughout the paper. Unless stated otherwise, all sets under consideration are assumed to be finite.

1.1 | n -th Powerset

The n -th powerset of a set S is obtained by applying the powerset operator successively n times[1]. Thus it collects progressively deeper families of subsets and captures higher-order grouping structures within S . Formal definitions follow.

Definition 1 (Universal Set). Let U be a *universal set* containing every object under consideration. All other sets in this work are assumed to satisfy $S \subseteq U$.

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Definition 2 (Base Set). [2] A *base set* S is any subset of U from which more complex constructions originate:

$$S \subseteq U.$$

All subsequent constructions such as powersets or higher-order powersets begin with elements of S .

Definition 3 (Power Set). [3] The *power set* of S , denoted $\mathcal{P}(S)$, is the set of all subsets of S :

$$\mathcal{P}(S) = \{ A \mid A \subseteq S \}.$$

Definition 4 (n -th Power Set). [1, 4] Define recursively the n -th *power set* of S by

$$\mathcal{P}^{(1)}(S) = \mathcal{P}(S), \quad \mathcal{P}^{(n+1)}(S) = \mathcal{P}(\mathcal{P}^{(n)}(S)), \quad n \geq 1.$$

Similarly, the n -th *nonempty power set* of S is

$$\mathcal{P}^{*(1)}(S) = \mathcal{P}(S) \setminus \{\emptyset\}, \quad \mathcal{P}^{*(n+1)}(S) = \mathcal{P}(\mathcal{P}^{*(n)}(S)) \setminus \{\emptyset\}.$$

Example 5 (Hierarchical Weather Forecasting with Higher-Order Power Sets). **Base level** ($n = 0$). Consider the set of *elementary weather states*

$$S = \{\text{Sunny, Cloudy, Rainy, Snowy}\}.$$

First power set ($n = 1$). The ordinary powerset

$$\mathcal{P}^{(1)}(S) = \mathcal{P}(S)$$

lists every possible *hourly condition mix*. For instance $\{\text{Sunny, Windy}\}$ (if “Windy” is encoded as a modifier) or $\{\text{Rainy, Cloudy}\}$ (light showers).

Second power set ($n = 2$). Each element of

$$\mathcal{P}^{(2)}(S) = \mathcal{P}(\mathcal{P}^{(1)}(S))$$

is a *set of hourly mixes*, i.e. a **single-day forecast scenario**. An example day could be

$$D_{\text{Mon}} = \{\{\text{Sunny}\}, \{\text{Cloudy}\}, \{\text{Rainy, Cloudy}\}\},$$

meaning: morning sunshine, afternoon overcast, evening showers.

Third power set ($n = 3$). An element of

$$\mathcal{P}^{(3)}(S) = \mathcal{P}(\mathcal{P}^{(2)}(S))$$

is a *set of day-scenarios*, hence a **weekly ensemble forecast**. For instance

$$\mathcal{W} = \{D_{\text{Mon}}, D_{\text{Tue}}, D_{\text{Wed}}\}$$

captures three alternative five-day outlooks produced by different numerical weather models.

Interpretation in practice.

- (1) $\mathcal{P}^{(1)}(S)$ (**hour layer**) — possible blends of basic states within an hour.
- (2) $\mathcal{P}^{(2)}(S)$ (**day layer**) — sequences (as sets) of hourly blends forming one-day scenarios.
- (3) $\mathcal{P}^{(3)}(S)$ (**week layer**) — collections of daily scenarios representing multi-model or probabilistic weekly forecasts.

Higher orders ($n > 3$) could encode, for example, portfolios of weekly ensembles maintained by different meteorological centres. Thus the n -th powerset provides a natural algebraic hierarchy for structuring increasingly coarser—but information-rich—levels of real-world weather prediction.

2 | Plithogenic Set for Weather Prediction

In this section, we examine the Plithogenic Set for Weather Prediction. Plithogenic Sets are advanced mathematical structures that generalize fuzzy [5, 6] or neutrosophic sets [7, 8] by integrating contradictory attributes and multiple membership degrees [9, 10, 11, 12].

Definition 6 (Plithogenic Set). [13, 9] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v .
- $pdf : P \times Pv \rightarrow [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)*¹
- $pCF : Pv \times Pv \rightarrow [0, 1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

- (1) *Reflexivity of Contradiction Function:*

$$pCF(a, a) = 0$$

- (2) *Symmetry of Contradiction Function:*

$$pCF(a, b) = pCF(b, a)$$

Example 7 (Weather Prediction using Precipitation Intensity). Needless to say, Weather Prediction estimates future meteorological conditions, including temperature, precipitation, and wind, using various advanced data-driven predictive models (cf.[14]). Precipitation Intensity in weather quantifies the rate of rainfall or snowfall, indicating classifications like light, moderate, or heavy precipitation conditions (cf.[15, 16]). Consider a weather prediction scenario for a given day. Let

$$S = \{\text{Sunny, Partly Cloudy, Rainy}\}$$

be the universal set of weather conditions, and let

$$P = \{\text{Sunny, Partly Cloudy, Rainy}\}$$

be the set of predicted weather conditions. Define the attribute

$$v = \text{Precipitation Intensity,}$$

with the set of possible values

$$Pv = \{\text{None, Light, Moderate, Heavy}\}.$$

The Degree of Appurtenance Function $pdf : P \times Pv \rightarrow [0, 1]$ is defined as:

$$\begin{aligned} pdf(\text{Sunny, None}) &= 0.95, & pdf(\text{Sunny, Light}) &= 0.05, \\ pdf(\text{Partly Cloudy, None}) &= 0.50, & pdf(\text{Partly Cloudy, Light}) &= 0.50, \\ pdf(\text{Rainy, Moderate}) &= 0.70, & pdf(\text{Rainy, Heavy}) &= 0.30, \end{aligned}$$

with all other combinations assumed to be zero.

¹It is important to note that the definition of the Degree of Appurtenance Function varies across different papers. Some studies define this concept using the power set, while others simplify it by avoiding the use of the power set [12]. The author has consistently defined the Classical Plithogenic Set without employing the power set.

The Degree of Contradiction Function $pCF : Pv \times Pv \rightarrow [0, 1]$ is defined by:

$$\begin{aligned} pCF(\text{None}, \text{None}) &= 0, & pCF(\text{None}, \text{Light}) &= 0.2, \\ pCF(\text{None}, \text{Moderate}) &= 0.5, & pCF(\text{None}, \text{Heavy}) &= 1.0, \\ pCF(\text{Light}, \text{Light}) &= 0, & pCF(\text{Light}, \text{Moderate}) &= 0.3, \\ pCF(\text{Light}, \text{Heavy}) &= 0.8, & pCF(\text{Moderate}, \text{Moderate}) &= 0, \\ pCF(\text{Moderate}, \text{Heavy}) &= 0.6, & pCF(\text{Heavy}, \text{Heavy}) &= 0. \end{aligned}$$

Note that pCF is symmetric, i.e., $pCF(a, b) = pCF(b, a)$ for all $a, b \in Pv$.

Example 8 (Weather Prediction using Temperature Levels). Consider a weather prediction scenario for a given day. Let

$$S = \{\text{Cool}, \text{Mild}, \text{Hot}\}$$

be the universal set of temperature conditions, and let

$$P = \{\text{Cool}, \text{Mild}, \text{Hot}\}$$

be the set of predicted temperature conditions. Define the attribute

$$v = \text{Temperature Level},$$

with the set of possible values

$$Pv = \{\text{Low}, \text{Medium}, \text{High}\}.$$

The Degree of Appurtenance Function $pdf : P \times Pv \rightarrow [0, 1]$ is defined as:

$$\begin{aligned} pdf(\text{Cool}, \text{Low}) &= 0.8, & pdf(\text{Cool}, \text{Medium}) &= 0.2, \\ pdf(\text{Mild}, \text{Medium}) &= 0.7, & pdf(\text{Mild}, \text{High}) &= 0.3, \\ pdf(\text{Hot}, \text{High}) &= 0.9, & pdf(\text{Hot}, \text{Medium}) &= 0.1, \end{aligned}$$

with all other combinations assumed to be zero.

The Degree of Contradiction Function $pCF : Pv \times Pv \rightarrow [0, 1]$ is defined by:

$$\begin{aligned} pCF(\text{Low}, \text{Low}) &= 0, & pCF(\text{Low}, \text{Medium}) &= 0.4, \\ pCF(\text{Low}, \text{High}) &= 0.9, & pCF(\text{Medium}, \text{Medium}) &= 0, \\ pCF(\text{Medium}, \text{High}) &= 0.5, & pCF(\text{High}, \text{High}) &= 0. \end{aligned}$$

Again, note that pCF is symmetric.

3 | HyperPlithogenic Set for Weather Prediction

In this section, we examine the HyperPlithogenic Set for Weather Prediction. HyperPlithogenic Sets are advanced higher-level generalizations of Plithogenic Sets, seamlessly incorporating multiple attributes, each with varying membership degrees, and contradictions [17, 18].

Definition 9 (HyperPlithogenic Set). [19] Let X be a non-empty set, and let A be a set of attributes. For each attribute $v \in A$, let Pv be the set of possible values of v . A *HyperPlithogenic Set HPS* over X is defined as:

$$HPS = (P, \{v_i\}_{i=1}^n, \{Pv_i\}_{i=1}^n, \{\tilde{pdf}_i\}_{i=1}^n, pCF)$$

where:

- $P \subseteq X$ is a subset of the universe.
- For each attribute v_i , Pv_i is the set of possible values.
- For each attribute v_i , $\tilde{pdf}_i : P \times Pv_i \rightarrow \tilde{P}([0, 1]^s)$ is the *Hyper Degree of Appurtenance Function (HDAF)*, assigning to each element $x \in P$ and attribute value $a_i \in Pv_i$ a set of membership degrees.

- $pCF : (\bigcup_{i=1}^n Pv_i) \times (\bigcup_{i=1}^n Pv_i) \rightarrow [0, 1]^t$ is the *Degree of Contradiction Function (DCF)*.

Example 10 (HyperPlithogenic Set for a Single Weather Forecast). Let X be the set of weather forecast events and consider a single forecast event $P = \{x\}$ for a given day. Define two attributes:

$$\begin{aligned} v_1 &= \text{Temperature} \quad \text{with} \quad Pv_1 = \{\text{Low, Moderate, High}\}, \\ v_2 &= \text{Precipitation} \quad \text{with} \quad Pv_2 = \{\text{None, Light, Heavy}\}. \end{aligned}$$

The Hyper Degree of Appurtenance Functions (HDAFs) for the forecast x are defined as follows:

$$\begin{aligned} \tilde{p}df_1(x, \text{Low}) &= \{0.25\}, \quad \tilde{p}df_1(x, \text{Moderate}) = \{0.65\}, \quad \tilde{p}df_1(x, \text{High}) = \{0.10\}, \\ \tilde{p}df_2(x, \text{None}) &= \{0.80\}, \quad \tilde{p}df_2(x, \text{Light}) = \{0.15\}, \quad \tilde{p}df_2(x, \text{Heavy}) = \{0.05\}. \end{aligned}$$

The Degree of Contradiction Function pCF is defined on the union

$$Pv_1 \cup Pv_2 = \{\text{Low, Moderate, High, None, Light, Heavy}\}.$$

For example, we may set:

$$\begin{aligned} pCF(\text{Low, Moderate}) &= 0.3, \quad pCF(\text{Low, High}) = 0.7, \quad pCF(\text{Moderate, High}) = 0.4, \\ pCF(\text{None, Light}) &= 0.2, \quad pCF(\text{None, Heavy}) = 0.8, \quad pCF(\text{Light, Heavy}) = 0.5, \\ pCF(\text{Moderate, None}) &= 0.4, \quad pCF(\text{High, Heavy}) = 0.3. \end{aligned}$$

By definition, $pCF(a, a) = 0$ for all $a \in Pv_1 \cup Pv_2$.

Thus, the HyperPlithogenic Set for this weather forecast is given by:

$$HPS = (\{x\}, \{\text{Temperature, Precipitation}\}, \{Pv_1, Pv_2\}, \{\tilde{p}df_1, \tilde{p}df_2\}, pCF).$$

Example 11 (HyperPlithogenic Set for Multiple Weather Forecasts). Let X be the set of weather forecast events and consider two forecast events $P = \{x_1, x_2\}$ for two consecutive days. Define the attributes as:

$$\begin{aligned} v_1 &= \text{Temperature} \quad \text{with} \quad Pv_1 = \{\text{Low, Moderate, High}\}, \\ v_2 &= \text{Precipitation} \quad \text{with} \quad Pv_2 = \{\text{None, Light, Heavy}\}. \end{aligned}$$

The Hyper Degree of Appurtenance Functions for the forecast events are given by:

For x_1 :

$$\begin{aligned} \tilde{p}df_1(x_1, \text{Low}) &= \{0.30\}, \quad \tilde{p}df_1(x_1, \text{Moderate}) = \{0.50\}, \quad \tilde{p}df_1(x_1, \text{High}) = \{0.20\}, \\ \tilde{p}df_2(x_1, \text{None}) &= \{0.75\}, \quad \tilde{p}df_2(x_1, \text{Light}) = \{0.20\}, \quad \tilde{p}df_2(x_1, \text{Heavy}) = \{0.05\}. \end{aligned}$$

For x_2 :

$$\begin{aligned} \tilde{p}df_1(x_2, \text{Low}) &= \{0.15\}, \quad \tilde{p}df_1(x_2, \text{Moderate}) = \{0.60\}, \quad \tilde{p}df_1(x_2, \text{High}) = \{0.25\}, \\ \tilde{p}df_2(x_2, \text{None}) &= \{0.65\}, \quad \tilde{p}df_2(x_2, \text{Light}) = \{0.30\}, \quad \tilde{p}df_2(x_2, \text{Heavy}) = \{0.05\}. \end{aligned}$$

The Degree of Contradiction Function pCF is defined on

$$Pv_1 \cup Pv_2 = \{\text{Low, Moderate, High, None, Light, Heavy}\},$$

and we set:

$$\begin{aligned} pCF(\text{Low, Moderate}) &= 0.3, \quad pCF(\text{Low, High}) = 0.7, \quad pCF(\text{Moderate, High}) = 0.4, \\ pCF(\text{None, Light}) &= 0.2, \quad pCF(\text{None, Heavy}) = 0.8, \quad pCF(\text{Light, Heavy}) = 0.5, \\ pCF(\text{Moderate, None}) &= 0.4, \quad pCF(\text{High, Heavy}) = 0.3. \end{aligned}$$

Again, $pCF(a, a) = 0$ for all $a \in Pv_1 \cup Pv_2$.

Thus, the HyperPlithogenic Set for these weather forecasts is:

$$HPS = (\{x_1, x_2\}, \{\text{Temperature, Precipitation}\}, \{Pv_1, Pv_2\}, \{\tilde{p}df_1, \tilde{p}df_2\}, pCF).$$

4 | n -SuperHyperPlithogenic Set for Weather Prediction

In this section, we examine the n -SuperHyperPlithogenic Set for Weather Prediction. An n -SuperHyperPlithogenic Set is an extension of the HyperPlithogenic Set, recursively incorporating multi-level membership structures and contradictions across nested subsets [19, 18].

Definition 12 (n -SuperHyperPlithogenic Set). [19] Let X be a non-empty set, and let $V = \{v_1, v_2, \dots, v_n\}$ be a set of attributes, each associated with a set of possible values P_{v_i} . An n -SuperHyperPlithogenic Set ($SHPS_n$) is defined recursively as:

$$SHPS_n = (P_n, V, \{P_{v_i}\}_{i=1}^n, \{\tilde{pdf}_i^{(n)}\}_{i=1}^n, pCF^{(n)}),$$

where:

- $P_1 \subseteq X$, and for $k \geq 2$,

$$P_k = \tilde{\mathcal{P}}(P_{k-1}),$$

represents the k -th nested family of non-empty subsets of P_1 .

- For each attribute $v_i \in V$, P_{v_i} is the set of possible values of the attribute v_i .
- For each k -th level subset P_k , $\tilde{pdf}_i^{(n)} : P_n \times P_{v_i} \rightarrow \tilde{\mathcal{P}}([0, 1]^s)$ is the *Hyper Degree of Appurtenance Function (HDAF)*, assigning to each element $x \in P_n$ and attribute value $a_i \in P_{v_i}$ a subset of $[0, 1]^s$.
- $pCF^{(n)} : \bigcup_{i=1}^n P_{v_i} \times \bigcup_{i=1}^n P_{v_i} \rightarrow [0, 1]^t$ is the *Degree of Contradiction Function (DCF)*, satisfying:
 - (1) Reflexivity: $pCF^{(n)}(a, a) = 0$ for all $a \in \bigcup_{i=1}^n P_{v_i}$,
 - (2) Symmetry: $pCF^{(n)}(a, b) = pCF^{(n)}(b, a)$ for all $a, b \in \bigcup_{i=1}^n P_{v_i}$.
- s and t are positive integers representing the dimensions of the membership degrees and contradiction degrees, respectively.

Example 13 (2-SuperHyperPlithogenic Set for Weather Prediction). Let $X = \{D_1, D_2\}$ be the set of forecast days, and define

$$P_1 = X = \{D_1, D_2\}.$$

Then, the second-level nested family is given by

$$P_2 = \tilde{\mathcal{P}}(P_1) = \{\{D_1\}, \{D_2\}, \{D_1, D_2\}\}.$$

Consider two attributes:

$$v_1 = \text{Temperature} \quad \text{with} \quad P_{v_1} = \{\text{Low}, \text{Moderate}, \text{High}\},$$

$$v_2 = \text{Precipitation} \quad \text{with} \quad P_{v_2} = \{\text{None}, \text{Light}, \text{Heavy}\}.$$

Assume the membership degree space is one-dimensional, i.e., $[0, 1]^1$. The Hyper Degree of Appurtenance Functions at the second level,

$$\tilde{pdf}_i^{(2)} : P_2 \times P_{v_i} \rightarrow \tilde{\mathcal{P}}([0, 1]),$$

are defined as follows.

For v_1 (Temperature):

$$\begin{aligned}
\tilde{p}df_1^{(2)}(\{D_1\}, \text{Low}) &= \{0.70\}, & \tilde{p}df_1^{(2)}(\{D_1\}, \text{Moderate}) &= \{0.25\}, \\
\tilde{p}df_1^{(2)}(\{D_1\}, \text{High}) &= \{0.05\}, & & \\
\tilde{p}df_1^{(2)}(\{D_2\}, \text{Low}) &= \{0.10\}, & \tilde{p}df_1^{(2)}(\{D_2\}, \text{Moderate}) &= \{0.60\}, \\
\tilde{p}df_1^{(2)}(\{D_2\}, \text{High}) &= \{0.30\}, & & \\
\tilde{p}df_1^{(2)}(\{D_1, D_2\}, \text{Low}) &= \{0.40\}, & \tilde{p}df_1^{(2)}(\{D_1, D_2\}, \text{Moderate}) &= \{0.50\}, \\
\tilde{p}df_1^{(2)}(\{D_1, D_2\}, \text{High}) &= \{0.10\}. & &
\end{aligned}$$

For v_2 (Precipitation):

$$\begin{aligned}
\tilde{p}df_2^{(2)}(\{D_1\}, \text{None}) &= \{0.85\}, & \tilde{p}df_2^{(2)}(\{D_1\}, \text{Light}) &= \{0.10\}, \\
\tilde{p}df_2^{(2)}(\{D_1\}, \text{Heavy}) &= \{0.05\}, & & \\
\tilde{p}df_2^{(2)}(\{D_2\}, \text{None}) &= \{0.40\}, & \tilde{p}df_2^{(2)}(\{D_2\}, \text{Light}) &= \{0.50\}, \\
\tilde{p}df_2^{(2)}(\{D_2\}, \text{Heavy}) &= \{0.10\}, & & \\
\tilde{p}df_2^{(2)}(\{D_1, D_2\}, \text{None}) &= \{0.60\}, & \tilde{p}df_2^{(2)}(\{D_1, D_2\}, \text{Light}) &= \{0.30\}, \\
\tilde{p}df_2^{(2)}(\{D_1, D_2\}, \text{Heavy}) &= \{0.10\}. & &
\end{aligned}$$

The Degree of Contradiction Function at the second level,

$$pCF^{(2)} : (P_{v_1} \cup P_{v_2}) \times (P_{v_1} \cup P_{v_2}) \rightarrow [0, 1],$$

is defined by specifying:

$$\begin{aligned}
pCF^{(2)}(\text{Low}, \text{Moderate}) &= 0.30, & pCF^{(2)}(\text{Low}, \text{High}) &= 0.70, \\
pCF^{(2)}(\text{Moderate}, \text{High}) &= 0.40, & & \\
pCF^{(2)}(\text{None}, \text{Light}) &= 0.20, & pCF^{(2)}(\text{None}, \text{Heavy}) &= 0.90, \\
pCF^{(2)}(\text{Light}, \text{Heavy}) &= 0.50, & & \\
pCF^{(2)}(\text{Low}, \text{None}) &= 0.10, & pCF^{(2)}(\text{Moderate}, \text{Light}) &= 0.20, \\
pCF^{(2)}(\text{High}, \text{Heavy}) &= 0.30. & &
\end{aligned}$$

Symmetry and reflexivity hold, i.e., $pCF^{(2)}(a, a) = 0$ and $pCF^{(2)}(a, b) = pCF^{(2)}(b, a)$ for all a, b .

Thus, the 2-SuperHyperPlithogenic Set for weather prediction is given by:

$$SHPS_2 = \left(P_2, \{\text{Temperature}, \text{Precipitation}\}, \{P_{v_1}, P_{v_2}\}, \{\tilde{p}df_1^{(2)}, \tilde{p}df_2^{(2)}\}, pCF^{(2)} \right).$$

Example 14 (3-SuperHyperPlithogenic Set for Weather Prediction). Let $X = \{D_1, D_2, D_3\}$ be the set of forecast days, and set

$$P_1 = X = \{D_1, D_2, D_3\}.$$

The second-level nested family is

$$P_2 = \tilde{\mathcal{P}}(P_1) = \{\{D_1\}, \{D_2\}, \{D_3\}, \{D_1, D_2\}, \{D_1, D_3\}, \{D_2, D_3\}, \{D_1, D_2, D_3\}\}.$$

The third-level nested family is then

$$P_3 = \tilde{\mathcal{P}}(P_2),$$

and we consider a specific element

$$X^* = \left\{ \{D_1\}, \{D_1, D_2\}, \{D_1, D_2, D_3\} \right\} \in P_3.$$

Let the set of attributes be

$$V = \{v_1, v_2, v_3\},$$

with

$$v_1 = \text{Temperature} \quad \text{and} \quad P_{v_1} = \{\text{Low}, \text{Moderate}, \text{High}\},$$

$$v_2 = \text{Humidity} \quad \text{and} \quad P_{v_2} = \{\text{Dry}, \text{Normal}, \text{Wet}\},$$

$$v_3 = \text{Precipitation} \quad \text{and} \quad P_{v_3} = \{\text{None}, \text{Light}, \text{Heavy}\}.$$

Assume $[0, 1]^1$ as the membership degree space. The Hyper Degree of Appurtenance Functions at the third level,

$$\tilde{p}df_i^{(3)} : P_3 \times P_{v_i} \rightarrow \tilde{\mathcal{P}}([0, 1]),$$

are defined for the chosen X^* as follows.

For v_1 (Temperature):

$$\tilde{p}df_1^{(3)}(X^*, \text{Low}) = \{0.45\},$$

$$\tilde{p}df_1^{(3)}(X^*, \text{Moderate}) = \{0.40\},$$

$$\tilde{p}df_1^{(3)}(X^*, \text{High}) = \{0.15\}.$$

For v_2 (Humidity):

$$\tilde{p}df_2^{(3)}(X^*, \text{Dry}) = \{0.30\},$$

$$\tilde{p}df_2^{(3)}(X^*, \text{Normal}) = \{0.50\},$$

$$\tilde{p}df_2^{(3)}(X^*, \text{Wet}) = \{0.20\}.$$

For v_3 (Precipitation):

$$\tilde{p}df_3^{(3)}(X^*, \text{None}) = \{0.70\},$$

$$\tilde{p}df_3^{(3)}(X^*, \text{Light}) = \{0.25\},$$

$$\tilde{p}df_3^{(3)}(X^*, \text{Heavy}) = \{0.05\}.$$

The Degree of Contradiction Function at the third level,

$$pCF^{(3)} : (P_{v_1} \cup P_{v_2} \cup P_{v_3}) \times (P_{v_1} \cup P_{v_2} \cup P_{v_3}) \rightarrow [0, 1],$$

is defined by:

$$pCF^{(3)}(\text{Low}, \text{Moderate}) = 0.25, \quad pCF^{(3)}(\text{Low}, \text{High}) = 0.65,$$

$$pCF^{(3)}(\text{Moderate}, \text{High}) = 0.35,$$

$$pCF^{(3)}(\text{Dry}, \text{Normal}) = 0.20, \quad pCF^{(3)}(\text{Dry}, \text{Wet}) = 0.50,$$

$$pCF^{(3)}(\text{Normal}, \text{Wet}) = 0.30,$$

$$pCF^{(3)}(\text{None}, \text{Light}) = 0.15, \quad pCF^{(3)}(\text{None}, \text{Heavy}) = 0.80,$$

$$pCF^{(3)}(\text{Light}, \text{Heavy}) = 0.45,$$

$$pCF^{(3)}(\text{Low}, \text{Dry}) = 0.10, \quad pCF^{(3)}(\text{Moderate}, \text{Light}) = 0.20,$$

$$pCF^{(3)}(\text{High}, \text{Heavy}) = 0.30.$$

Again, symmetry and reflexivity hold:

$$pCF^{(3)}(a, a) = 0 \quad \text{and} \quad pCF^{(3)}(a, b) = pCF^{(3)}(b, a) \quad \forall a, b.$$

Thus, the 3-SuperHyperPlithogenic Set for weather prediction is expressed as:

$$SHPS_3 = \left(P_3, \{\text{Temperature}, \text{Humidity}, \text{Precipitation}\}, \{P_{v_1}, P_{v_2}, P_{v_3}\}, \{\tilde{p}df_1^{(3)}, \tilde{p}df_2^{(3)}, \tilde{p}df_3^{(3)}\}, pCF^{(3)} \right).$$

5 | Conclusion and Future Work

This study has demonstrated that the three interconnected frameworks—*Plithogenic Sets*, *HyperPlithogenic Sets*, and *SuperHyperPlithogenic Sets*—can all be applied effectively to weather forecasting.

As future directions, we intend to construct weather-prediction models formulated in the language of *Graphs* [20, 21], *Hypergraphs* [22, 23, 24, 25], and *SuperHypergraphs* [26, 27, 28, 29]. We also look forward to advances in computational experiments and algorithm design for Plithogenic, HyperPlithogenic, and SuperHyperPlithogenic Sets themselves.

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available due to privacy-preserving constraints but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in this research.

Ethical Approval

This article does not contain any studies involving human participants or animals conducted by any of the authors.

References

- [1] Florentin Smarandache. Superhyperstructure & neutrosophic superhyperstructure, 2024. Accessed: 2024-12-01.
- [2] Takaaki Fujita. Superhypergraph neural networks and plithogenic graph neural networks: Theoretical foundations. *arXiv preprint arXiv:2412.01176*, 2024.
- [3] Florentin Smarandache. The cardinal of the m-powerset of a set of n elements used in the superhyperstructures and neutrosophic superhyperstructures. *Systems Assessment and Engineering Management*, 2:19–22, 2024.
- [4] Adel Al-Odhari. Neutrosophic power-set and neutrosophic hyper-structure of neutrosophic set of three types. *Annals of Pure and Applied Mathematics*, 31(2):125–146, 2025.
- [5] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.
- [6] Hongxing Li and Vincent C Yen. *Fuzzy sets and fuzzy decision-making*. CRC press, 1995.
- [7] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Single valued neutrosophic graphs. *Journal of New theory*, (10):86–101, 2016.
- [8] S. Onar. A note on neutrosophic soft set over hyperalgebras. *Symmetry*, 16(10):1288, 2024.
- [9] Florentin Smarandache. *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study, 2018.
- [10] P Sathya, Nivetha Martin, and Florentine Smarandache. Plithogenic forest hypersoft sets in plithogenic contradiction based multi-criteria decision making. *Neutrosophic Sets and Systems*, 73:668–693, 2024.
- [11] S Gomathy, D Nagarajan, S Broumi, and M Lathamaheswari. *Plithogenic sets and their application in decision making*. Infinite Study, 2020.
- [12] S Sudha, Nivetha Martin, and Florentin Smarandache. *Applications of Extended Plithogenic Sets in Plithogenic Sociogram*. Infinite Study, 2023.
- [13] Fazeelat Sultana, Muhammad Gulistan, Mumtaz Ali, Naveed Yaqoob, Muhammad Khan, Tabasam Rashid, and Tauseef Ahmed. A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, 14(10):13139–13159, 2023.
- [14] Peter Bauer, Alan Thorpe, and Gilbert Brunet. The quiet revolution of numerical weather prediction. *Nature*, 525(7567):47–55, 2015.
- [15] Angeline G Pendergrass, Flavio Lehner, Benjamin M Sanderson, and Yangyang Xu. Does extreme precipitation intensity depend on the emissions scenario? *Geophysical Research Letters*, 42(20):8767–8774, 2015.
- [16] A Michael, J Schmidt, W Enke, Th Deutschländer, and G Malitz. Impact of expected increase in precipitation intensities on soil loss-results of comparative model simulations. *Catena*, 61(2-3):155–164, 2005.
- [17] Takaaki Fujita. Hyperplithogenic cubic set and superhyperplithogenic cubic set. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, page 79, 2025.
- [18] Florentin Smarandache. Hyperuncertain, superuncertain, and superhyperuncertain sets/logics/probabilities/statistics. *Critical Review*, XIV, 2017.
- [19] Takaaki Fujita. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*. Biblio Publishing, 2025.
- [20] Jonathan L Gross, Jay Yellen, and Mark Anderson. *Graph theory and its applications*. Chapman and Hall/CRC, 2018.
- [21] Reinhard Diestel. *Graph theory*, volume 173. Springer Nature, 2025.
- [22] Alain Bretto. *Hypergraph theory. An introduction*. Mathematical Engineering. Cham: Springer, 1, 2013.
- [23] Claude Berge. *Hypergraphs: combinatorics of finite sets*, volume 45. Elsevier, 1984.
- [24] Muhammad Akram and Gulfam Shahzadi. Hypergraphs in m-polar fuzzy environment. *Mathematics*, 6(2):28, 2018.

- [25] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558–3565, 2019.
- [26] Takaaki Fujita and Arkan A Ghaib. Toward a unified theory of brain hypergraphs and symptom hypernetworks in medicine and neuroscience. *Advances in Research*, 26(3):522–565, 2025.
- [27] Mohammed Alqahtani. Intuitionistic fuzzy quasi-supergraph integration for social network decision making. *International Journal of Analysis and Applications*, 23:137–137, 2025.
- [28] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.
- [29] Masoud Ghods, Zahra Rostami, and Florentin Smarandache. Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties. *Neutrosophic Sets and Systems*, 50:480–487, 2022.

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