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Philosophical Meditations on Crispness vs. Fuzziness: A Review

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Abstract

What is mathematical language? How is this language used to explain and analyze the components of the material universe, both ontology and epistemology? This paper highlights how mathematical models, as ontological and cognitive models, grow and develop within their mathematical environment. Through a series of articles in the light of contemporary mathematical concepts. In this article, we will present the concept of crispness versus fuzziness by comparing crisp-set and fuzzy-set.

Keywords: Crispness; Crisp-Set; Fuzziness; Fuzzy-Set.

1 | Introduction

This article has two aims: one immediate and related to our previous works [1-5], which is to present a simplified picture of the development of the concepts of Crisp-Sets, Fuzzy-Sets, Intuitionistic-Sets, and Neutrosophic-Sets through the idea and model of the mathematical object. The other, more distant, represents a future vision related to the philosophy of mathematics, its nature, and role, through a comparison of the mathematical concepts that have emerged over the past sixty years and before, and the nature of the philosophical concepts associated with them. This is a challenging task, given the realities and circumstances around us, and our ability to understand various disciplines within mathematical and philosophical subjects. Anyway, the class of Crisp-Set represents a dividing line between two intellectual trends in the field of mathematical thought. The first trend represents the doctrine/ideology of mathematical certainty, while the other represents the doctrine/ideology of mathematical ambiguity and indeterminacy. This distinction will open the door wide in the future for the philosophy of mathematics in particular, and the interaction of philosophy, logic, and mathematics in general, to explore the essence of these differences and their roles and functions in serving scientific purposes in the age of artificial intelligence, big-data, and information networks. We addressed section two for Crisp-Sets as a key point for the new mathematical concepts, and the third section for Fuzzy-Sets. Intuitionistic-Sets and Neutrosophic-Sets, we will discuss in future work.

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2 | Crisp-Set as a Key Point For The New Mathematical Concepts

In the late 19th century, Georg Cantor developed Set Theory, and the concept of a "Crisp-Set" is a particular object in Set Theory. Moreover, the term "Crisp-Set" emphasizes classical sets that are clear-cut, unambiguous, non-fuzzy, determinate, and certain. In this section, we provide an overview of "Crisp-Set" and prove some results related to it.

Definition 2.1 [6] A Set (or Class) is a well-defined collection of distinct objects. An object in a set is called an element or member of that set. This concept was introduced by G. Cantor (1845-1918), who is considered the father of set theory, or, more accurately, the genesis of Set Theory.

Notation. We will denote any set consisting of two elements by 2. Let us say $2 = \{0,1\}$.

Definition 2.2 A Crisp-Set (CS) is a bilateral structure class, written

$$A_{CS} = \{(u, \mu_A(u)) : u \in U\} \quad (2.1)$$

And it consists of:

- A universal set/ discourse U is a domain of μ_A ,
- A subset set A of a set U ($A \subseteq U$) is a predicate of μ_A ,
- Set of two elements $2 = \{0,1\}$ is a codomain of μ_A ,
- $\mu_A: U \mapsto 2 = \{0,1\}$ is called the characteristic function of $u \in U$ to a set A in U , or the degree of membership function of $u \in U$ to a set A in U , or the truth of the predicate with the membership function property

$$\mu_A(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \in A^c = U - A \end{cases} \quad (2.2)$$

Notation. We will use $\hat{A}$ (read $\hat{A}$ hat) instead of A_{CS} for shortening. Figure 1 represents the degree of membership function of a set A in U .

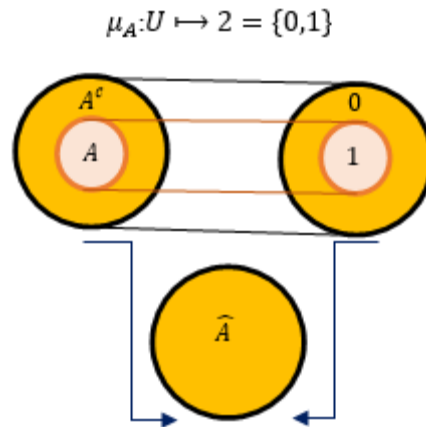


Figure 1. Crisp-Set $\hat{A} = \{(u, \mu_A(u)) : u \in U \text{ \& } \mu_A(u) \in 2\}$.

Here, μ_A is called the one-valued characteristic function or single-valued characteristic function.

Observation. In the language of Set Theory, if U is a universal set, then there exist the following facts, $\emptyset, U \subseteq U$. And it is proved according to concepts defined in Set Theory. So, if $\emptyset \subseteq U$, then:

$$\mu_{\emptyset}(x) = \begin{cases} 1, & \text{if } u \in \emptyset \\ 0, & \text{if } u \in \emptyset^c = U \end{cases} \quad (2.3)$$

Also, if $U \subseteq U$, then:

$$\mu_U(u) = \begin{cases} 1, & \text{if } u \in U \\ 0, & \text{if } u \in U^c = \emptyset \end{cases} \quad (2.4)$$

Definition 2.3 A CrispSet Complement (CSC) is a bilateral structure, written

$$A_{CS}^c = \{(u, \mu_{A^c}(u)) : u \in U\} \quad (2.5)$$

And it consists of:

- i. A universal set/ discourse U is a domain of μ_{A^c} ,
- ii. A subset set A^c of a set U ($A^c \subseteq U$) is a predicate of μ_{A^c}
- iii. Set of two elements $2 = \{0,1\}$ is a codomain of μ_{A^c} , and
- iv. $\mu_{A^c}: U \mapsto 2 = \{0,1\}$ is called the characteristic function of $u \in U$ to a set A^c in U , or the degree of membership function of $u \in U$ to a set A^c in U , or the truth of the predicate, and it has the membership function property:

$$\mu_{A^c}(u) = \begin{cases} 1, & \text{if } x \in A^c \\ 0, & \text{if } x \in A^{cc} = A \end{cases} \quad (2.6)$$

Notation. We will use $\widehat{A}^c$ (read $\widehat{A}^c$ complement hat) instead of A_{CS}^c for brevity.

Example 2.1 Let $U = \{a, b, c, d, e\}$, if $A = \{a, b\} \subset U$, then $\mu_A(a) = \mu_A(b) = 1$, and

$\mu_A(c) = \mu_A(d) = \mu_A(e) = 0$. The entity of the Crisp-Set $\widehat{A}$ Looks like:

$$\begin{aligned} \widehat{A} &= \{(u, \mu_A(u)) : \forall u \in U\} \\ &= \{(a, 1), (b, 1), (c, 0), (d, 0), (e, 0)\} \\ \widehat{A} &\subseteq U \times \{0,1\}. \end{aligned} \quad (2.7)$$

A Crisp-Set $\widehat{A}$ is a subset of ordered pairs of the Cartesian product $U \times \{0,1\}$. In addition, we have, $A^c = \{c, d, e\}$, hence $\mu_{A^c}(c) = \mu_{A^c}(d) = \mu_{A^c}(e) = 1$ and $\mu_A(a) = \mu_A(b) = 0$. So, the entity of the

Crisp-Set complement $\widehat{A}^c$ becomes:

$$\begin{aligned} \widehat{A}^c &= \{(u, \mu_A(u)) : \forall u \in U\} \\ &= \{(a, 0), (b, 0), (c, 1), (d, 1), (e, 1)\} \\ \widehat{A}^c &\subseteq U \times \{0,1\} \end{aligned} \quad (2.8)$$

Moreover, $\widehat{A} \cup \widehat{A}^c = U \times \{0,1\}$ and $\widehat{A} \cap \widehat{A}^c = \emptyset$. Thus $\widehat{A}$ and $\widehat{A}^c$ are partiation of $U \times \{0,1\}$.

Example 2.2 Let $\mathbb{N}$ be a set of all natural numbers, and $\widehat{A} = \{x \in \mathbb{N} : x \geq 10\}$, then we have

$$\mu_A(u) = \begin{cases} 1, & \text{if } u \geq 10 \\ 0, & \text{if } u < 10 \end{cases} \quad (2.9)$$

so in this case, for instance, we have $\mu_A(8) = 0$ and $\mu_A(20) = 1$.

Definition 2.4 The Empty Crisp-Set is defined by:

$$\widehat{A} = \emptyset \quad \text{iff } \mu_A(u) = 0, \forall u \in U. \quad (2.10)$$

Definition 2.5 The Universal Crisp-Set is defined by:

$$\widehat{A} = U \quad \text{iff } \mu_A(u) = 1, \forall u \in U. \quad (2.11)$$

Note that. The Empty Crisp-Set and Universal Crisp-Set are models of constant functions.

The graph presentation of Crisp-Set $\hat{A}$ can be represented by lattice points in 2-dimensions. The next theorem gives us some operations and relations on the class of Crisp-Set.

Theorem 2.1 Let $\hat{A}$ and $\hat{B}$ be two crisp subsets of U . Then:

$$\bullet \text{ Crisp-Set Inclusion: } (\forall u \in U), (\hat{A} \subseteq \hat{B}) \Leftrightarrow (\mu_{A \cap B} = \mu_A) \quad (2.12)$$

$$\bullet \text{ Crisp-Set Equality: } (\forall u \in U), (\hat{A} = \hat{B}) \Leftrightarrow (\mu_A = \mu_B) \quad (2.13)$$

$$\bullet \text{ Crisp-Set Union: } (\forall u \in U), (\hat{A} \cup \hat{B}) \Leftrightarrow (\mu_{A \cup B} = \mu_A + \mu_B - \mu_{A \cap B}) \quad (2.14)$$

$$\bullet \text{ Crisp-Set Intersection: } (\forall u \in U), (\hat{A} \cap \hat{B}) \Leftrightarrow (\mu_{A \cap B} = \mu_A \cdot \mu_B) \quad (2.15)$$

$$\bullet \text{ Crisp-Set Difference: } (\forall u \in U), (\hat{A} - \hat{B}) \Leftrightarrow (\mu_{A-B} = \mu_A - \mu_{A \cap B}) \quad (2.16)$$

Proof.

- Suppose that $\hat{A} \subseteq \hat{B}$ and $u \in A \cap B$, then $\mu_{A \cap B} = 1$ and $\mu_A = 1$. Also, if $u \notin A \cap B$, then $u \notin A$ and we have $\mu_{A \cap B} = 0$ and $\mu_A = 0$. Therefore, in both cases $\mu_{A \cap B} = \mu_A$. Conversely, Assume that $\mu_{A \cap B} = \mu_A$ and $(u, \mu_A(u)) \in \hat{A}$. Since $A \cap B \subset B$, then $\mu_{A \cap B}(u) \subset \mu_B(u)$, hence $\mu_A(u) \subset \mu_B(u)$, therefore $(u, \mu_A(u)) \in \hat{B}$. Thus $\hat{A} \subseteq \hat{B}$.
- $\hat{A} = \hat{B} \Leftrightarrow (u, \mu_A(u)) = (u, \mu_B(u)), \forall u \in U, \text{Dom}(\mu_A) = \text{Dom}(\mu_B) \text{ and } \text{Ran}(\mu_A) = \text{Ran}(\mu_B)$
 - $\Leftrightarrow \mu_A(u) = \mu_B(u) \Leftrightarrow \mu_A = \mu_B$.

1. Let $u \in A \cup B$, then $\mu_{A \cup B}(u) = 1$. So we have three cases:
 - i. If $u \in A$ and $u \notin B$, then $\mu_A + \mu_B - \mu_{A \cap B} = 1 + 0 - 0 = 1 = \mu_{A \cup B}(u)$.
 - ii. If $u \notin A$ and $u \in B$, then $\mu_A + \mu_B - \mu_{A \cap B} = 0 + 1 - 0 = 1 = \mu_{A \cup B}(u)$.
 - iii. If $u \in A$ and $u \in B$, $\mu_A + \mu_B - \mu_{A \cap B} = 1 + 1 - 1 = 2 - 1 = 1 = \mu_{A \cup B}(u)$. Now, if $u \notin A \cup B$, then $\mu_{A \cup B}(u) = 0$ and also $u \notin A$ and $u \notin B$, so $\mu_A + \mu_B - \mu_{A \cap B} = 0 + 0 - 0 = 0 = \mu_{A \cup B}(u)$.
2. If $u \in A \cap B$, then $\mu_{A \cap B}(u) = 1$, and $\mu_A \cdot \mu_B = 1 \cdot 1 = 1$. if $x \notin A \cap B$, then $\mu_{A \cap B}(u) = 0$ and so $\mu_A \cdot \mu_B = 0 \cdot 0 = 0$.
3. If $u \in A - B$, then $\mu_{A-B}(u) = 1$, and we have $u \in A$ and $u \notin B$, then $\mu_A - \mu_{A \cap B} = 1 - 0 = 1$. When $u \notin A - B$, then $\mu_{A-B}(u) = 0$, and also, we have $u \notin A$ or $u \in B$, now if $u \notin A$, then $\mu_A - \mu_{A \cap B} = 0 - 0 = 0$.
4. If $u \in A \cap B$, then . ■

The next theorem gives us the important property that connects or links the PowerSet $P(U)$ and the set 2^U .

Theorem 2.2 Let U be a universal set, and $P(U)$ be the power set of U . Consider 2^U is the set of all functions from a set U into the set 2. Then $P(U) \approx 2^U$. I.e., they are equivalent sets.

Proof. Consider the set $2 = \{0,1\}$, and $2^U = \{f|f: U \mapsto 2\}$ is a set of all functions from a set U into the set 2. Define a binary relation $\Psi: P(U) \mapsto 2^U$ as follows:

$$\Psi = \{(A, f) \in P(U) \times 2^U : f = \mu_A\} \quad (2.17)$$

Where μ_A is the characteristic function of A in U or the degree of membership function of A in U which is given by:

$$\mu_A(u) = \begin{cases} 1, & \text{if } u \in A \\ 0, & \text{if } u \in A^c = U - A \end{cases} \quad (2.18)$$

First, we need to show that Ψ is a function. Suppose that $A \in P(U)$. Since,

$$A \in P(U) \Rightarrow \exists \mu_A \in 2^U \ni (A, \mu_A), \forall A \in P(U). \quad (2.19)$$

Now, assume that

$$((A, f_1) \in \Psi \wedge (A, f_2) \in \Psi) \Rightarrow ((f_1 = \mu_A) \wedge (f_2 = \mu_A)) \Rightarrow f_1 = f_2. \quad (2.20)$$

Therefore, Ψ is a function. Second, we want to show that Ψ is a one-to-one. Suppose that, $A_1, A_2 \in P(U)$ such that $\Psi(A_1) = \Psi(A_2)$. Since,

$$\Psi(A_1) = \mu_{A_1} \wedge \Psi(A_2) = \mu_{A_2} \Rightarrow \mu_{A_1} = \mu_{A_2}, \text{ hence} \quad (2.21)$$

$$A_1 = \{x \in U: \mu_{A_1}(x) = 1\} = \{x \in U: \mu_{A_2}(x) = 1\} = A_2, \quad (2.22)$$

Therefore $A_1 = A_2$ and Ψ is a one-to-one function. Third, to prove that Ψ is onto (surjective). Assume that $f \in 2^U$, then $f: U \mapsto 2 = \{0, 1\}$. Let $f^{-1}(1) = A$ be the inverse image of zero equal to A , hence,

$A \in P(U)$, but $\Psi(A) = \mu_A$.

$$\text{Since } f(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \in U - A \end{cases} \quad (2.23)$$

Hence $\Psi(A) = \mu_A = f$, and consequently, Ψ is onto function, we conclude that Ψ is a bijective function ■.

So far, we have reviewed and discussed the concepts of CrispSet, CrispSet Complement, Empty-CrispSet, Universal-CrispSet, CrispSet-Equality, CrispSet-Union, CrispSet-Intersection, and CrispSet-Differences on the previous arguments.

Of course! The role of the characteristics function reflects many real-world situations that require responding between two different choices. Therefore, it represents one example that illustrates the nature of a universal language, through which existence can be expressed in a single and neutral manner, namely, the language of mathematics.

Definition 2.5. [7] The semantic (or meaning) of propositional logic consists of truth valuations. A valuation (or truth assignment or interpretation) v in the language L is a function from the set of simple statement letters into the set $\{T, F\}$. I.e.,

$$v(A) = \begin{cases} T, & \text{if } A \text{ is true} \\ F, & \text{if } A \text{ is false} \end{cases} \quad (2.24)$$

which satisfies the following conditions:

$$\left. \begin{aligned} 1. & v(\neg A) \neq v(A), \\ 2. & v(A \wedge B) = T \leftrightarrow v(A) = v(B) = T, \\ 3. & v(A \vee B) = F \leftrightarrow v(A) = v(B) = F \\ 4. & v(A \vee B) = F \leftrightarrow v(A) = v(B) = F \end{aligned} \right\} \quad (2.25)$$

By comparing Definition 2.1 and Theorem 2.2 with Definition 2.5, we can register the following observations:

- $\mu_A(x)$ and $v(A)$ have the same structure as a piecewise function.
- $\mu_A(x)$ as a mathematical entity, it belongs to the field of Set Theory. While,
- $v(A)$ as a logical entity, it belongs to the field of Mathematical Logic or Philosophy.
- Domain of $\mu_A(x)$ is a well-defined collection of objects, but the Domain of $v(A)$ is a well-defined collection of all simple propositions. So, they have different domains.

- The codomain (= Range) of $\mu_A(x)$, and $v(A)$, they seemed the same when put $T = 1$ and $F = 0$. However, they have different meanings, specifically membership (truth, falsehood).
- The patterns of both structures seem identical, but they have different interpretations.

3 | Zadeh and Fuzzy Sets

In 1965, Zadeh published the first article on Fuzzy Sets. In his introduction, he inquired about the criteria absent for classical membership functions, when we need to express certain issues of the real physical world. He justified the use of Fuzzy-Sets as an extension of Crisp-Sets. From here starts the new ideology of uncertainty versus the ideology of certainty. Moreover, terms such as uncertainty, ambiguity, and ill-defined are necessary where the boundaries between their members and non-members are not sharp and distinct. The exposition of this material is based on [8-16]. Let us now reflect on the doctrine of ambiguity through the statements of scholars in the cited sources as follows:

1. "The classes of objects encountered in the real physical world do not have precisely defined criteria of membership. (Zadeh, p.338)." He illustrated this by classifying animals and distinguishing between a bacterium and a fish-star to determine whether it belongs to the class of animals or not. Zadeh here refers to the boundaries between their members and non-members, which are not sharp. Of course! Someone can ask Is the class of animals well-defined in the language of set theory? According to Zadeh, this class is ill-defined.
2. "The process and progress of knowledge unfolds into two stages: an attempt to know the character of the world and a subsequent attempt to know the character of knowledge itself. " And they said, " Our problems are seen in terms of decision, management, and prediction; solutions are seen in terms of faster access to more information and of increased aid in analyzing, understanding and utilizing the information that is available and in coping with the information that is not..., large amounts of information coupled with large amounts of uncertainty, taken together constitute the ground of many of our problems today: complexity (Kilr & Folger, p.1)." In fact, Kilr and Folger point to the role of knowledge, its advancement, and its implications for decision-making and predictions in many organizations and major economies, which justifies the importance of fuzzy-sets and their role in finding solutions and dealing with such issues. In addition, every era in human history has a cognitive advantage that distinguishes it from others, and our current era is characterized by the era of big data, informatics, artificial intelligence, and information overload.
3. Why do we need fuzzy mathematics? Singh said, "How many seeds in a pile constitute a heap? Indeed, ' heap' is a fuzzy concept as it a fuzzy phenomenon that does not exist a unique number number can represented a heap. If we consider a natural phenomenon, say "rain," then again, we arrive at the difficulty of representing it in a precise manner, as it can rain with varying intensity, from a light sprinkle to moderate rain, or heavy rain, or a torrential downpour—because, again, rain is a fuzzy concept (Singh, p.ix)." As a matter of fact, we have quite a lot of issues to understand and attempt to get over. One of them, for instance, relates to Natural Linguistics, such as the English Language. Let us claim that the mathematical language is the language of our Universe. As such, it is obliged to explain both existential and cognitive beings. Now, consider that the English Language is one of the existential and cognitive beings. We will imagine that there are classes, such as a class of words associated with a class of clear meanings, and a class of fuzzy words associated with fuzzy meanings. Let $FS = \{s: s \text{ is a fuzzy – sentence}\}$ be the class of all fuzzy sentences that constitute the class of fuzzy words in English. The fuzziness (ambiguity) of vague predicates originates from a lack of objective definitions in natural language for the class of fuzzy words associated with the class of fuzzy meanings of those words. Of course, this represents a fundamental problem at the level of natural language, and it is up to linguists to find solutions to it—if any exist—but since this is the case, and there are scientific phenomena related to this separation, how can they be explained and decisions made regarding it? Hence, the interaction of mathematics, as a universal language, with

natural languages to express these issues in order to meet the necessity of scientific need and serve the knowledge society. When we deal with ambiguous sentences, for example, this person is (...), the weather is (...), or the seeds constituted a (...), and the class of fuzzy words:

$FW = \{\text{tall, short, smart, beautiful, kind, generous, stringy, arrogant, humble, sunny, cloudy, heavy, light, heap, etc., ...}\}$. Here, the mathematical language is the most effective tool for describing neutral language in terms of the degree of fuzzy membership function, thereby addressing our scientific issues. Of course, different people may have different standards or criteria for what the meaning of the words in a class of FW depends on their perception of the subjective or objective of fuzzy words.

Definition 3.1 [15] Let U be a universal set and $A \subseteq U$. A **FuzzySet** (FS) is a bilateral structure, written

$$A_{FS} = \{(u, \mu_A(u)) : u \in U\} \quad (3.1)$$

And it consists of:

- i. A universal set/ discourse U is a domain of μ_A ,
- ii. A subset set A of a set $U (A \subseteq U)$ is a predicate of μ_A ,
- iii. Interval set $I = [0,1]$ is a codomain of μ_A , and
- iv. $\mu_A: U \mapsto [0,1]$ is called the partial-truth function of A f in U , or the degree of membership function of A in U , with the following membership function property:

$$(\forall u \in U) \Rightarrow (0 \leq \mu_A(u) \leq 1) \quad (3.2)$$

From Eq. (3.2) we have:

$$v_A(u) = 1 - \mu_A(u) \quad (3.3)$$

Eq. (3.3) is called the degree of non-membership function of $u \in U$ to set A .

Notation.

- We will $\bar{A}$ (read $\bar{A}$ bar) instead of A_{FS} for shortening.
- A fuzzy-set $\bar{A}$ is a vague boundary compared to a crisp set $\bar{A}$.
- μ_A is called the one-valued membership function or single-valued membership function.

Figure 2 represents the degree of membership function of A in U .

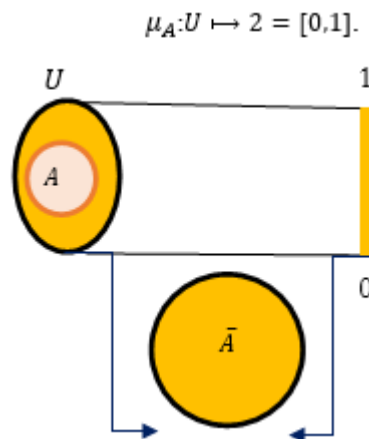


Figure 2. Fuzzy-Set $\bar{A} = \{(u, \mu_A(u)) : u \in U \text{ \& } \mu_A(u) \in [0,1]\}$.

Here, μ_A is called the one-valued membership function or single-valued membership function.

Definition 3.2 The Empty Fuzzy-Set is defined by: $\bar{A} = \emptyset$ iff $\mu_A(u) = 0, \forall u \in U$. The Empty Fuzz-Set is a model of a constant function and represents no presence unless is called the partial presence.

Definition 3.3 The Universal Fuzzy-Set is defined by: $\bar{A} = U$ iff $\mu_A(u) = 1, \forall u \in U$. Additionally, Universal Fuzzy-Set is a model of a constant function and interprets full presence as the absence of partial presence.

Definition 3.4 [15] (Operations and Relations on a FuzzySet). Let $\bar{A}$ and $\bar{B}$ be two Fuzzy subsets of U , then for all

$u \in U$, we have the following:

- Fuzzy-Set Inclusion:

$$\bar{A} \subseteq \bar{B} \text{ iff } \mu_A(u) \leq \mu_B(u), \quad (3.4)$$

- Fuzzy-Set Equality:

$$\bar{A} = \bar{B} \text{ iff } \mu_A(u) = \mu_B(u), \quad (3.5)$$

- Fuzzy-Set Complement:

$$\bar{A}^c = \left\{ \begin{array}{l} (u, \mu_{A^c}(u)): u \in U \\ \& \\ \mu_{A^c}(u) = 1 - \mu_A(u) \end{array} \right\} \quad (3.6)$$

- Fuzzy-Set Union:

$$\bar{A} \cup \bar{B} = \left\{ \begin{array}{l} (u, \mu_{A \cup B}(u)): u \in U \\ \& \\ \mu_{A \cup B}(u) = \max\{\mu_A(u), \mu_B(u)\} \end{array} \right\} \quad (3.7)$$

- Fuzzy-Set Intersection:

$$\bar{A} \cap \bar{B} = \left\{ \begin{array}{l} (u, \mu_{A \cap B}(u)): u \in U \\ \& \\ \mu_{A \cap B}(u) = \inf\{\mu_A(u), \mu_B(u)\} \end{array} \right\} \quad (3.8)$$

- Fuzzy-Set Difference:

$$\bar{A} - \bar{B} = \bar{A} \cap \bar{B}^c. \quad (3.9)$$

Example 3.1. Let $U = \mathbb{R}$ be the set of real numbers. Define the set of numbers close to any real number a by membership function: $\mu_A(x) = \frac{1}{(1+10(x-a)^2)}, \forall a \in \mathbb{R}$, where $\mu_A: \mathbb{R} \rightarrow [0,1]$. We illustrate this example by a particular case. Consider $\mu_A: U = [-10:0.1:10] \rightarrow [0,1]$. The set of numbers close to 0 is then shown in Figure 3.

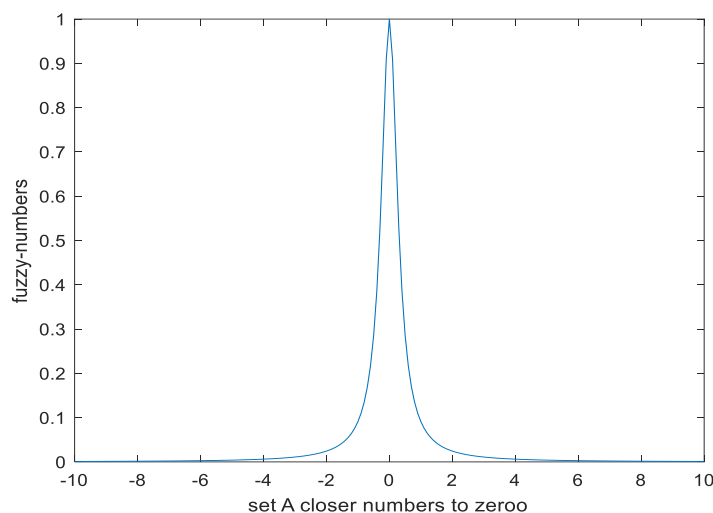


Figure 3. Fuzzy-Set $\bar{A} = \{u \in U: u \text{ is coser to } 0, \& \mu_A(u) \in [0,1]\}$.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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