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Logarithmic Ratio-Type Estimator for Finite Population Mean under Neutrosophic Framework

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Abstract

In the classical statistical framework, a precise and determinate dataset is used to estimate the population mean when auxiliary variables are known. In cases where the data is unclear or expressed in ranges, such as with daily city temperatures or stock prices, classical statistics fail to perform well in such situations. In these cases, neutrosophic statistics provide significantly greater precision. The primary objective of this study is to propose a neutrosophic estimator that minimizes the mean squared error (MSE) in estimating unknown finite population mean, particularly in cases where classical statistics fall short due to data ambiguity and indeterminacy. The concept of neutrosophic statistics was pioneered by Florentin Smarandache. It is a generalization of classical statistics that addresses ambiguous, unclear, vague, and indeterminate data. In this study, we present a neutrosophic logarithmic ratio-type estimator that uses known auxiliary information to estimate the neutrosophic mean of a study variable. The proposed estimators' bias and MSE are analyzed through a first-order approximation. Based on the MSE and percent relative efficiency (PRE) criteria, the proposed estimator shows improved performance over existing neutrosophic estimators. The proposed estimator's performance is validated through empirical studies, with its practical relevance demonstrated using real-life neutrosophic datasets from medical product sales and marketing. In addition, a simulation study was also carried out, showing that our proposed estimator performs better than those discussed in the previous studies.

Keywords: Classical Statistics; Neutrosophic Statistics; Logarithmic Ratio-Type Estimator; Auxiliary Information; Mean Squared Error (MSE); Percentage Relative Efficiency (PRE).

1 | Introduction

The objective of sampling theory is often to derive highly precise estimates of the study variable. Over the years, statisticians have come to appreciate the role of auxiliary information in improving estimation accuracy and have adopted it extensively. The choice of estimator is influenced by the correlation between the study and auxiliary variables. Specifically, ratio estimators are applied when there is a positive relationship between the study and auxiliary variables, while product estimators are chosen in the case of a negative relationship. As a result of ongoing innovation and in-depth statistical assessment, these estimators are seen as reliable and effective methods for population estimation in survey sampling. By incorporating auxiliary information, ratio, and product estimators achieve greater accuracy and efficiency. To enhance the efficiency and precision of

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several estimators, characteristics of auxiliary variables such as kurtosis and coefficients of variation are considered. Integrating the above information into the estimation process can yield more accurate and effective estimators tailored to the data's intricate nature and fluctuations. The development and enhancement of various estimation techniques, including ratio, product, and exponential estimators, have significantly contributed to improving the estimation of population means in sample surveys. Ratio estimators play a vital role in estimation theory by assuming a proportional relationship exists between the study and auxiliary variables. Using auxiliary data, these estimators derive a ratio between the relevant study variable and its corresponding auxiliary variable. Incorporating both the sample data and auxiliary information, the resulting estimator enhances the precision of the population mean estimate. Ratio estimators reflect both the inherent variability and the core association between variables, offering deep insights into population characteristics. Similar to ratio estimators, product estimators perform estimation by applying a multiplicative relationship between observed values and auxiliary information. These estimators bring a novel approach to estimating the population mean by modeling the connection between the main variable and its supportive factor. In situations involving multiplicative interactions between variables, product estimators prove useful because of their adaptable nature. Exponential estimators utilize exponential transformations of auxiliary variables, offering a unique and broadened approach to estimation [1].

In classical statistics, numerous estimation methods have been devised based on single as well as multiple auxiliary variables. [2] was the first to utilize auxiliary data to improve the precision of the ratio estimation technique. A ratio estimator is applied when the auxiliary variable is positively correlated with the study variable. The product method of estimation, introduced by [3] in 1964, was designed as a parallel to the ratio estimator. The product method of estimation is used when a negative correlation is observed between the study and auxiliary variables. The regression estimator provides more accurate results than the ratio and product estimators when the regression line does not pass through the origin. [4] introduced exponential ratio-product estimators that incorporate auxiliary information through second-order approximation techniques. [5] introduced a set of exponential ratio estimators that utilize two auxiliary variables to estimate the finite population mean. Exponential imputation approaches for estimating the population mean under the assumption of completely random missing data were suggested by [6].

Classical statistics operates on datasets where all values are crisp and the measurements involve no ambiguity. But in many situations, the data exhibits feature of uncertainty, and ambiguous interpretation. To tackle such ambiguity, L.A. Zadeh pioneered fuzzy logic as his first contribution [7]. In everyday scenarios, we often deal with indeterminate data, for example, temperature readings, decision-making systems, weather predictions, economic trends, etc. In these situations, it becomes essential to develop new methods for managing indeterminate data. To resolve uncertainty and vagueness in data, fuzzy logic is often applied, but it lacks the capacity to address indeterminacy. When faced with this kind of situation, methods based on neutrosophic logic are ideal for managing both unpredictability and indeterminacy. Florentin Smarandache proposed neutrosophy, an extension of fuzzy logic that considers both randomness and indeterminacy [8]. Neutrosophic statistics address scenarios with data uncertainty, unlike classical statistics, which are applied when the data is clear and deterministic [9]. When there is no indeterminacy, neutrosophic statistics behaves just like classical statistics, reflecting its role as an expansion of the classical model [9].

Neutrosophic statistics is effective in addressing various real-world challenges, including the study of traffic accidents, [10]. A goodness-of-fit method grounded in neutrosophic statistics was developed by [11]. Neutrosophic statistics, as described in [12], deals with situations where the sample size isn't precisely known. A more sophisticated version of fuzzy sets is the complex fuzzy set, and its broader generalization is known as the complex neutrosophic set. Under the framework of sampling theory, [13] addressed the estimation of the population mean in the face of data indeterminacy and lack of certainty. An almost unbiased estimator for determining the population mean using neutrosophic information was suggested by [14]. In an effort to enhance population mean estimation, [15] proposed a generalized method under the neutrosophic framework. In the past few years, the field of neutrosophic statistics has seen major contributions from various scholars. A neutrosophic ranked set sampling method and a corresponding generalized estimator for estimating the

population mean were presented by [16]. A neutrosophic exponential-type estimator utilizing both study and auxiliary variables was developed by [17] to address population mean estimation under uncertain conditions. Neutrosophic Hartley-Ross-type ratio estimators capable of estimating the population mean despite the presence of outliers and sensitive or non-sensitive variables were proposed by [18]. [19] proposed neutrosophic regression-cum-ratio estimators for estimating the population mean in uncertain environments, with applications in the field of medical science. A Neutrosophic Ranked Set Sampling approach aimed at estimating the population mean, with an application to demographic data was proposed by [20]. Using neutrosophic stratified sampling, [21] introduced new regression and ratio-based estimators and analyzed their statistical behavior using real-world climate data. To improve the estimation of the population mean, [22] proposed a new neutrosophic estimator that unites ratio-cum-product exponential methods and two auxiliary inputs. [23] introduced a family of neutrosophic estimators aimed at estimating the mean, using real-world data for application. A method for estimating the population mean using two neutrosophic auxiliary variables with uncertain information was proposed by [24]. An improved method for estimating the population mean using the known medians of two auxiliary variables under the neutrosophic setting was introduced by [25].

In our study, we have proposed a neutrosophic logarithmic ratio-type estimator to estimate finite population mean under uncertainty. This paper has been systematically arranged in the following manner: In Section 1, we provide the Introduction, followed by a discussion on neutrosophic statistical notation and symbols in Section 2. Section 3 presents an overview of several existing neutrosophic estimators. Section 4 introduces our proposed estimator and includes first-order derivations for the bias and MSE of the neutrosophic logarithmic ratio-type estimator. Section 5 presents a discussion on the theoretical efficiency comparison. Section 6 includes a real-world application drawn from the COVID-19 situation. Section 7 covers the simulation study. In Section 8, the results are presented and discussed in detail. In Section 9, we draw conclusions based on the research findings.

2 | Notation and Terminology

Various types of neutrosophic observations were presented, featuring numerical data that suggested the value might lie within an uncertain interval (a, b) [9]. Different techniques can be applied to express the interval form of a neutrosophic number. According to [13] and [15], such an interval value is denoted as $Z_N = Z_L + Z_U I_N$, with I_N belonging to the range $[I_L, I_U]$. In the neutrosophic variable Z_N , the symbols Z_L and Z_U denote its lower and upper limits, respectively, while I_N represents its indeterminacy degree, taking values between 0 and 1. Let the i^{th} observation of the neutrosophic variable y be represented as $y_N(i) \in [y_L, y_U]$. Similarly, the auxiliary variable x is denoted as $x_N(i) \in [x_L, x_U]$.

Let the neutrosophic sample means $\bar{y}_N(i) \in [\bar{y}_L, \bar{y}_U]$ and $\bar{x}_N(i) \in [\bar{x}_L, \bar{x}_U]$ represent the sample means of the neutrosophic variables y_N and x_N , respectively. Let $\bar{Y}_N(i)$ and $\bar{X}_N(i)$ be the population means of the study and auxiliary variables, respectively. The neutrosophic variables y_N and x_N , their respective coefficients of variation (CV) are denoted by $C_{yN} \in [C_{yL}, C_{yU}]$ and $C_{xN} \in [C_{xL}, C_{xU}]$. The neutrosophic correlation coefficient between study variable y_N and auxiliary variable x_N is given as $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$ respectively.

Neutrosophic relative errors for the sample means of the study and auxiliary variables y_N and x_N are represented as $e_{0N} = \frac{\bar{y}_N - \bar{Y}_N}{\bar{Y}_N}$ and $e_{1N} = \frac{\bar{x}_N - \bar{X}_N}{\bar{X}_N}$, respectively. To compute the bias and MSE of the estimator, we express it as follows,

$$\bar{y}_N = (1 + e_{0N})\bar{Y}_N \text{ and } \bar{x}_N = (1 + e_{1N})\bar{X}_N ,$$

$$E(e_{0N}) = E(e_{1N}) = 0 ;$$

$$E(e_{0N}^2) = f_N C_{yN}^2, E(e_{1N}^2) = f_N C_{xN}^2 \text{ and } E(e_{0N}e_{1N}) = f_N \rho_{yxN} C_{xN} C_{yN}.$$

where,

$e_{0N} \in [e_{0L}, e_{0U}]$, $e_{1N} \in [e_{1L}, e_{1U}]$ and $e_{0N}e_{1N} \in [e_{0L}e_{1L}, e_{0U}e_{1U}]$.

$f_N = \frac{N_N - n_N}{n_N N_N}$, where, $f_N \in [f_L, f_U]$.

$C_{yN}^2 = \frac{S_{yN}^2}{\bar{Y}_N^2}$, $C_{xN}^2 = \frac{S_{xN}^2}{\bar{X}_N^2}$, and $S_{yxN} = \rho_{yxN} C_{xN} C_{yN}$, where, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $S_{yN}^2 \in [S_{yL}^2, S_{yU}^2]$, $S_{xN}^2 \in [S_{xL}^2, S_{xU}^2]$, $S_{yxN} \in [S_{yxL}, S_{yxU}]$ and $\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$.

An illustration of the proposed study utilizing neutrosophic numbers is presented in the following Figure 1.

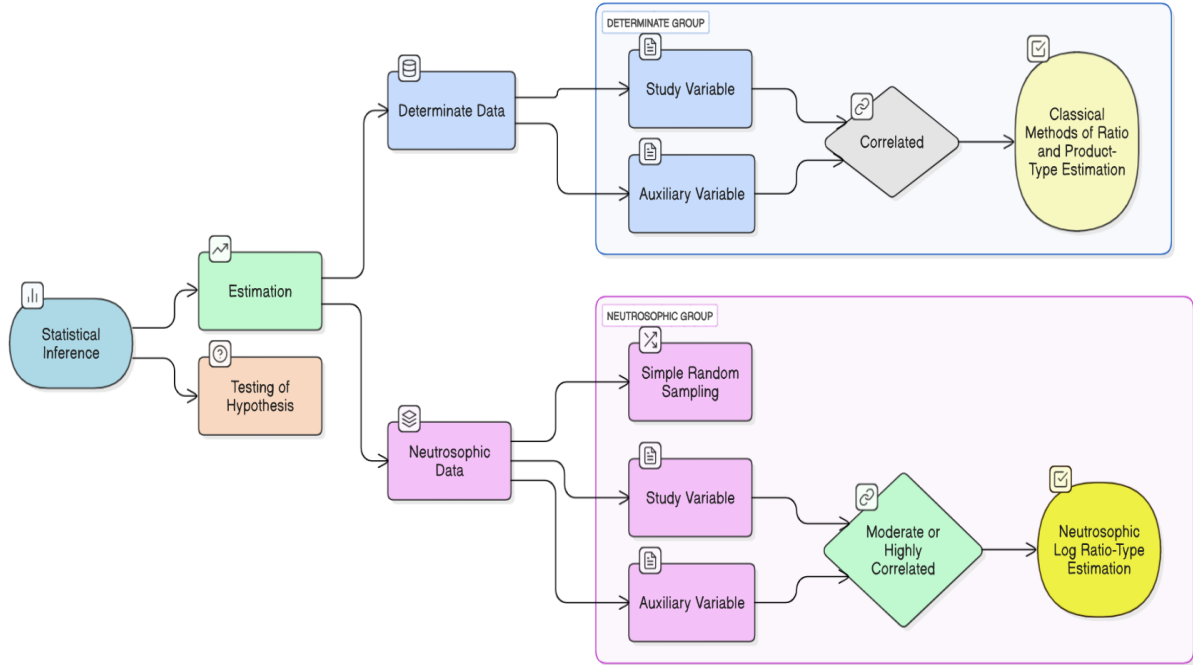


Figure 1. Flowchart of the parameter estimation process.

3 | Existing Neutrosophic Estimators

The neutrosophic sample mean is utilized to approximate the neutrosophic population mean ($\bar{Y}_N$) as,

$$t_{1N} = \bar{y}_N \quad (3.1)$$

The expression that defines the variance of the estimator (t_{1N}) is presented as follows,

$$V(t_{1N}) = f_N \bar{Y}_N^2 C_{yN}^2 \quad ; \text{where, } t_{1N} \in [t_{1L}, t_{1U}]. \quad (3.2)$$

Based on the work in [3], the neutrosophic product estimators are formulated as shown below,

$$t_{2N} = \bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N} \right) \quad (3.3)$$

The expressions for the bias and MSE of estimator (t_{2N}) are presented below,

$$\text{Bias}(t_{2N}) = f_N \bar{Y}_N \rho_{yxN} C_{yN} C_{xN} \quad (3.4)$$

$$\text{MSE}(t_{2N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + C_{xN}^2 + 2\rho_{yxN} C_{xN} C_{yN}) \quad ; \text{where, } t_{2N} \in [t_{2L}, t_{2U}]. \quad (3.5)$$

A Neutrosophic ratio estimator for estimating a finite population mean was introduced by [13], defined as follows,

$$t_{3N} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N} \right) \quad (3.6)$$

Using a first-order approximation, the bias and MSE for estimator (t_{3N}) are derived as follows,

$$\text{Bias}(t_{3N}) = f_N \bar{Y}_N (C_{xN}^2 - \rho_{yxN} C_{xN} C_{yN}) \quad (3.7)$$

$$\text{MSE}(t_{3N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + C_{xN}^2 - 2\rho_{yxN} C_{xN} C_{yN}) ; \text{ where, } t_{3N} \in [t_{3L}, t_{3U}]. \quad (3.8)$$

Motivated by [26], [13] suggested a neutrosophic ratio estimator that makes use of the coefficient of variation (CV) associated with the neutrosophic variable X as,

$$t_{4N} = \bar{y}_N \left(\frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}} \right) \quad (3.9)$$

Up to the first-order approximation, the bias and MSE of estimator (t_{4N}) are respectively as follows,

$$\text{Bias}(t_{4N}) = f_N \bar{Y}_N (\theta_{1N}^2 C_{xN}^2 - \theta_{1N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.10)$$

$$\text{MSE}(t_{4N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{1N}^2 C_{xN}^2 - 2\theta_{1N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.11)$$

$$\text{where, } \theta_{1N} = \frac{\bar{X}_N}{\bar{x}_N + C_{xN}} \text{ and } t_{4N} \in [t_{4L}, t_{4U}].$$

With reference to [1], [13] presented the following neutrosophic exponential ratio-type estimator as,

$$t_{5N} = \bar{y}_N \exp \left(\frac{\bar{X}_N - \bar{x}_N}{\bar{x}_N + \bar{X}_N} \right) \quad (3.12)$$

The expressions for the bias and MSE of estimator (t_{5N}) are provided below as,

$$\text{Bias}(t_{5N}) = f_N \bar{Y}_N \left(\frac{3}{8} C_{xN}^2 - \frac{1}{2} \rho_{yxN} C_{xN} C_{yN} \right) \quad (3.13)$$

$$\text{MSE}(t_{5N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} - \rho_{yxN} C_{xN} C_{yN} \right) ; \text{ where, } t_{5N} \in [t_{5L}, t_{5U}]. \quad (3.14)$$

With reference to [1], [27] suggested using the neutrosophic exponential product-type estimator outlined below as,

$$t_{6N} = \bar{y}_N \exp \left(\frac{\bar{x}_N - \bar{X}_N}{\bar{x}_N + \bar{X}_N} \right) \quad (3.15)$$

The corresponding expressions for its bias and MSE are given as follows,

$$\text{Bias}(t_{6N}) = f_N \bar{Y}_N \left(\frac{3}{8} C_{xN}^2 + \frac{1}{2} \rho_{yxN} C_{xN} C_{yN} \right) \quad (3.16)$$

$$\text{MSE}(t_{6N}) = f_N \bar{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} + \rho_{yxN} C_{xN} C_{yN} \right) ; \text{ where, } t_{6N} \in [t_{6L}, t_{6U}]. \quad (3.17)$$

A neutrosophic ratio estimator involving the population correlation coefficient was developed by [15],

$$t_{7N} = \bar{y}_N \left(\frac{\bar{X}_N + \rho_{yxN}}{\bar{x}_N + \rho_{yxN}} \right) \quad (3.18)$$

The expressions for the bias and MSE of estimator (t_{7N}) are provided below as,

$$\text{Bias}(t_{7N}) = f_N \bar{Y}_N (\theta_{2N}^2 C_{xN}^2 - \theta_{2N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.19)$$

$$\text{MSE}(t_{7N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{2N}^2 C_{xN}^2 - 2\theta_{2N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.20)$$

$$\text{where, } \theta_{2N} = \frac{\bar{X}_N}{\bar{x}_N + \rho_{yxN}} \text{ and } t_{7N} \in [t_{7L}, t_{7U}].$$

Inspired by [28], the neutrosophic ratio estimator (t_{8N}) was proposed using ratio estimators for the population mean based on the sample size selected from the population as,

$$t_{8N} = \bar{y}_N \left(\frac{\bar{X}_N + n_N}{\bar{x}_N + n_N} \right) \quad (3.21)$$

The bias and MSE of the neutrosophic ratio estimator (t_{8N}) are respectively expressed as follows,

$$\text{Bias}(t_{8N}) = f_N \bar{Y}_N (\theta_{3N}^2 C_{xN}^2 - \theta_{3N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.22)$$

$$\text{MSE}(t_{8N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{3N}^2 C_{xN}^2 - 2\theta_{3N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.23)$$

where, $\theta_{3N} = \frac{\bar{X}_N}{\bar{X}_N + n_N}$ and $t_{8N} \in [t_{8L}, t_{8U}]$.

Following the work of [29], [13] defined the neutrosophic modified exponential ratio estimator in the following way as,

$$t_{9N} = \bar{y}_N \exp \left(\frac{(a_N \bar{X}_N + b_N) - (a_N \bar{x}_n + b_N)}{(a_N \bar{X}_N + b_N) + (a_N \bar{x}_n + b_N)} \right) \quad (3.24)$$

where, a_N and b_N represent the auxiliary neutrosophic parameters.

For a first-order approximation, the neutrosophic exponential ratio estimator (t_{9N}) has the bias and MSE as follows,

$$\text{Bias}(t_{9N}) = f_N \bar{Y}_N \left(\frac{3}{8} \theta_{4N}^2 C_{xN}^2 - \frac{1}{2} \theta_{4N} \rho_{yxN} C_{xN} C_{yN} \right) \quad (3.25)$$

$$\text{MSE}(t_{9N}) = f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{4N}^2 C_{xN}^2 - 2\theta_{4N} \rho_{yxN} C_{xN} C_{yN}) \quad (3.26)$$

where, $\theta_{4N} = \frac{a_N \bar{X}_N}{2(a_N \bar{X}_N + b_N)}$ and $t_{9N} \in [t_{9L}, t_{9U}]$.

4 | Proposed Neutrosophic Estimator

This section presents improved neutrosophic logarithmic ratio-type estimator aimed at efficiently estimating the mean of a finite neutrosophic population. Drawing inspiration from the studies in [30] and [31], we developed an improved estimator that utilizes the known mean of an auxiliary variable under the neutrosophic approach. The following expression represents the proposed estimator as,

$$t_N^* = w_{1N} \left[\bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N} \right) \right] + w_{2N} \left[\frac{\bar{y}_N \ln \left(\frac{\bar{X}_N}{\bar{x}_N} \right)}{\bar{x}_N} \right] \bar{X}_N \quad (4.1)$$

where, w_{1N} and w_{2N} represents the scalars that to be identified in order to minimize the MSE of estimator (t_N^*).

The estimator t_N^* can be expressed with respect to error term e_{0N} and e_{1N} as follows:

$$t_N^* = w_{1N} \left[\bar{Y}_N (1 + e_{0N}) \left(\frac{\bar{X}_N}{\bar{X}_N (1 + e_{1N})} \right) \right] + w_{2N} \left[\frac{\bar{Y}_N (1 + e_{0N}) \ln \left(\frac{\bar{X}_N (1 + e_{1N})}{\bar{X}_N} \right)}{\bar{X}_N (1 + e_{1N})} \right] \bar{X}_N \quad (4.2)$$

$$t_N^* = w_{1N} [\bar{Y}_N (1 + e_{0N}) (1 + e_{1N})^{-1}] + w_{2N} [\bar{Y}_N (1 + e_{0N}) \ln((1 + e_{1N})) (1 + e_{1N})^{-1}] \quad (4.3)$$

$$t_N^* = w_{1N} [\bar{Y}_N (1 + e_{0N}) (1 - e_{1N} + e_{1N}^2)] + w_{2N} \left[\bar{Y}_N (1 + e_{0N}) \left(e_{1N} - \frac{e_{1N}^2}{2} \right) \right] [(1 - e_{1N} + e_{1N}^2)] \quad (4.4)$$

On simplification of the Eq. (4.4), we get,

$$t_N^* = w_{1N} [\bar{Y}_N (1 + e_{0N} - e_{1N} - e_{0N} e_{1N} + e_{1N}^2)] + w_{2N} \left[\bar{Y}_N \left(e_{1N} + e_{0N} e_{1N} - \frac{3}{2} e_{1N}^2 \right) \right] \quad (4.5)$$

Subtracting $\bar{Y}_N$ on each side of Eq. (4.5) leads to,

$$t_N^* - \bar{Y}_N = w_{1N} [\bar{Y}_N (1 + e_{0N} - e_{1N} - e_{0N} e_{1N} + e_{1N}^2)] + w_{2N} \left[\bar{Y}_N \left(e_{1N} + e_{0N} e_{1N} - \frac{3}{2} e_{1N}^2 \right) \right] - \bar{Y}_N \quad (4.6)$$

After applying expectations to both sides of Eq. (4.6) and replacing with corresponding expectation values, the bias of estimator t_N^* up to first-order approximation is calculated as,

$$\text{Bias}(t_N^*) = E(t_N^* - \bar{Y}_N) \quad (4.7)$$

$$\text{Bias}(t_N^*) = \bar{Y}_N \left[w_{1N} \{1 + f_N C_{xN}^2 - f_N \rho_{yxN} C_{xN} C_{yN}\} + w_{2N} \left\{ f_N \rho_{yxN} C_{xN} C_{yN} - \frac{3}{2} f_N C_{xN}^2 \right\} - 1 \right] \quad (4.8)$$

On squaring each side of Eq. (4.6) and simplifying to the first-order approximation, we obtain,

$$(t_N^* - \bar{Y}_N)^2 = \left(w_{1N} [\bar{Y}_N (1 + e_{0N} - e_{1N} - e_{0N} e_{1N} + e_{1N}^2)] + w_{2N} \left[\bar{Y}_N \left(e_{1N} + e_{0N} e_{1N} - \frac{3}{2} e_{1N}^2 \right) \right] - \bar{Y}_N \right)^2 \quad (4.9)$$

By taking expectation on both sides of Eq. (4.9), we get the following results,

$$\text{MSE}(t_N^*) = A_N w_{1N}^2 + B_N w_{2N}^2 + C_N + 2D_N w_{1N} w_{2N} - 2E_N w_{1N} - 2F_N w_{2N} \quad (4.10)$$

where,

$$A_N = \bar{Y}_N^2 (1 + f_N C_{yN}^2 + 3f_N C_{xN}^2 - 4f_N \rho_{yxN} C_{xN} C_{yN}).$$

$$B_N = \bar{Y}_N^2 f_N C_{xN}^2,$$

$$C_N = \bar{Y}_N^2.$$

$$D_N = \bar{Y}_N^2 \left(2f_N \rho_{yxN} C_{xN} C_{yN} - \frac{5}{2} f_N C_{xN}^2 \right).$$

$$E_N = \bar{Y}_N^2 \{1 + f_N C_{xN}^2 - f_N \rho_{yxN} C_{xN} C_{yN}\}.$$

$$F_N = \bar{Y}_N^2 \left(f_N \rho_{yxN} C_{xN} C_{yN} - \frac{3}{2} f_N C_{xN}^2 \right).$$

The optimum values for constants w_{1N} and w_{2N} that minimize the MSE of the proposed estimator (t_N^*) are determined as:

$$w_{1N(opt)} = \frac{B_N E_N - D_N F_N}{A_N B_N - D_N^2} \quad \text{and} \quad w_{2N(opt)} = \frac{A_N F_N - D_N E_N}{A_N B_N - D_N^2}.$$

The minimum MSE for estimator (t_N^*) under the optimum values of w_{1N} and w_{2N} is obtained as,

$$\text{MSE}_{min}(t_N^*) = \left[A_N \left(\frac{B_N E_N - D_N F_N}{A_N B_N - D_N^2} \right)^2 + B_N \left(\frac{A_N F_N - D_N E_N}{A_N B_N - D_N^2} \right)^2 + C_N + 2D_N \left(\frac{B_N E_N - D_N F_N}{A_N B_N - D_N^2} \right) \left(\frac{A_N F_N - D_N E_N}{A_N B_N - D_N^2} \right) - 2E_N \left(\frac{B_N E_N - D_N F_N}{A_N B_N - D_N^2} \right) - 2F_N \left(\frac{A_N F_N - D_N E_N}{A_N B_N - D_N^2} \right) \right] \quad (4.11)$$

$$\text{MSE}_{min}(t_N^*) = \left[\frac{2D_N E_N F_N - A_N F_N^2 - B_N E_N^2}{A_N B_N - D_N^2} + C_N \right] \quad (4.12)$$

$$\text{MSE}_{min}(t_N^*) = \left[\frac{P_N}{Q_N} + C_N \right] \quad (4.13)$$

where,

$$P_N = 2D_N E_N F_N - A_N F_N^2 - B_N E_N^2.$$

$$Q_N = A_N B_N - D_N^2.$$

5 | Theoretical Efficiency Comparison

In this section, a comparative study is carried out here between the newly proposed estimator and existing neutrosophic estimators of the population mean ($\bar{Y}_N$), employing neutrosophic auxiliary variable. The performance of the suggested estimator has been analyzed in terms of MSE, and the condition for it to exhibit greater efficiency than the existing estimator has been determined.

The proposed estimator t_N^* exhibits greater efficiency than estimator (t_{1N}) provided that the condition holds,

$$V(t_{1N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 C_{yN}^2 - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The proposed estimator t_N^* has a smaller MSE compared to estimator (t_{2N}) under the following condition,

$$MSE(t_{2N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + C_{xN}^2 + 2\rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

Under a certain restriction, the proposed estimator t_N^* performs better than estimator (t_{3N}) ,

$$MSE(t_{3N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + C_{xN}^2 - 2\rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The condition under which the proposed estimator t_N^* outperforms estimator (t_{4N}) is outlined as follows,

$$MSE(t_{4N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{1N}^2 C_{xN}^2 - 2\theta_{1N} \rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The proposed estimator t_N^* is superior to the exponential ratio estimator (t_{5N}) , under the specified condition,

$$MSE(t_{5N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} - \rho_{yxN} C_{xN} C_{yN} \right) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

Under a certain condition, the proposed estimator t_N^* performs better than estimator (t_{6N}) ,

$$MSE(t_{6N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 \left(C_{yN}^2 + \frac{C_{xN}^2}{4} + \rho_{yxN} C_{xN} C_{yN} \right) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The condition under which the proposed estimator t_N^* outperforms estimator (t_{7N}) is outlined as follows,

$$MSE(t_{7N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{2N}^2 C_{xN}^2 - 2\theta_{2N} \rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The proposed estimator t_N^* exhibits greater efficiency than estimator (t_{8N}) provided that the condition holds,

$$MSE(t_{8N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{3N}^2 C_{xN}^2 - 2\theta_{3N} \rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

The condition under which the proposed estimator t_N^* outperforms estimator (t_{9N}) is outlined as follows,

$$MSE(t_{9N}) - MSE_{min}(t_N^*) > 0 \text{ or,}$$

$$f_N \bar{Y}_N^2 (C_{yN}^2 + \theta_{4N}^2 C_{xN}^2 - 2\theta_{4N} \rho_{yxN} C_{xN} C_{yN}) - \left[\frac{P_N}{Q_N} + C_N \right] > 0 .$$

6 | Empirical Study

To illustrate the characteristics of our proposed neutrosophic estimators numerically, we employed two neutrosophic datasets related to product sales and medical marketing, sourced from [32]. Medical professionals collected networking and product sales data for two months before the lockdown (January 1 - March 1, 2021) and during the lockdown (September 1 - November 1, 2022). These indeterminate data consist

of two distinct sets - each containing customer networking and product sales variables, one representing the pre-pandemic period and other the post-pandemic period.

Dataset 1: We treat the pre-pandemic networking percentage as an auxiliary information, while the pre-pandemic sales percentage is considered as a study variable, i.e.,

X_N : Pre-pandemic networking percentage

Y_N : Pre-pandemic sales percentage

Dataset 2: In a similar manner, in the second dataset, the post-pandemic networking percentage is used as an auxiliary information, while the post-pandemic sales percentage is considered as the study variable. The study variable under neutrosophic framework is given by $Y_N \in [Y_L, Y_U]$, where Y_L indicating the lowest sales percentage from the pre- or post-pandemic period, and the corresponding neutrosophic auxiliary variable is $X_N \in [X_L, X_U]$. The parameters considered in this study are summarized in Table 1.

X_N : Post-pandemic networking percentage

Y_N : Post-pandemic sales percentage

Table 1. Descriptive statistic for datasets 1 and 2.

Parameters	Dataset 1	Dataset 2
N_N	[30, 30]	[30, 30]
n_N	[9, 9]	[9, 9]
$\bar{Y}_N$	[37.0033, 37.3266]	[34.7266, 40.040]
$\bar{X}_N$	[34.1600, 34.4500]	[34.7266, 34.9033]
S_{yN}	[12.3153, 12.3233]	[12.2704, 26.7004]
S_{xN}	[15.0789, 15.0891]	[15.2288, 15.2452]
C_{yN}	[0.3328, 0.3301]	[0.3533, 0.6668]
C_{xN}	[0.4414, 0.4380]	[0.4385, 0.4367]
ρ_{yxN}	[0.8776, 0.8775]	[0.8508, 0.1462]

Percentage Relative Efficiency (PRE) is a statistical tool used to compare the efficiency of two or more estimators involved in estimating a specific parameter. It is usually calculated as a percentage, based on the ratio between the variance of the estimator under consideration to that of a usual estimator. A higher PRE indicates a more efficient estimator, which yields more precise estimates with lower variability.

The following formula is used to compute the Percent Relative Efficiency (PRE) of the estimator as,

$$PRE = \frac{MSE(\bar{y}_N)}{MSE(estimator)} \times 100.$$

Table 2. MSEs and PREs value for proposed estimator and various existing estimators for dataset 1.

Estimator	MSE	I_N	PRE
t_{1N}	[11.7951, 11.8082]	[0, 0.0011]	[100, 100]
t_{2N}	[60.0029, 60.0948]	[0, 0.0015]	[19.6576, 19.6493]
t_{3N}	[5.0857, 5.1003]	[0, 0.0029]	[231.9272, 231.5202]
t_{4N}	[31.7638, 31.8282]	[0, 0.0020]	[37.1339, 37.0997]
t_{5N}	[3.2531, 3.2569]	[0, 0.0012]	[362.5786, 362.5587]
t_{6N}	[30.7117, 30.7541]	[0, 0.0014]	[38.4060, 38.3954]
t_{7N}	[4.7471, 4.7633]	[0, 0.0034]	[248.4722, 247.8971]
t_{8N}	[3.0603, 3.0755]	[0, 0.0049]	[385.4255, 383.9444]
t_{9N}	[3.2531, 3.2569]	[0, 0.0012]	[362.5786, 362.5587]
t_N^*	[2.6999, 2.7051]	[0, 0.0019]	[436.8796, 436.5178]

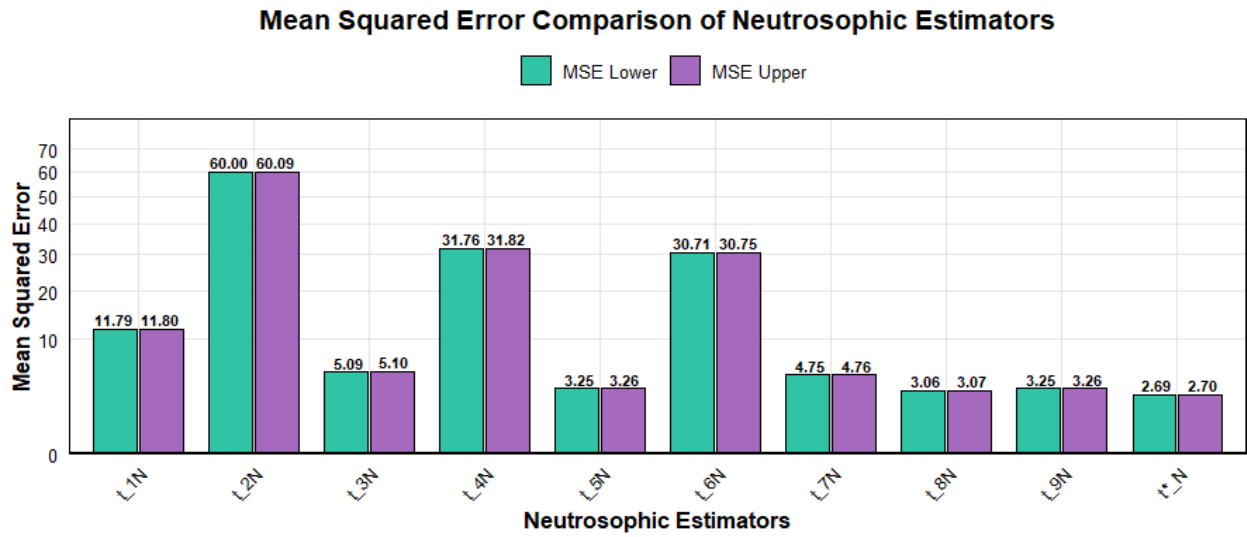


Figure 2. MSEs values for proposed estimator and various existing estimators for dataset 1.

Table 3. MSEs and PREs value for proposed estimator and various existing estimators for dataset 2.

Estimator	MSE	I_N	PRE
t_{1N}	[11.7076, 55.4415]	[0, 0.7888]	[100, 100]
t_{2N}	[54.4685, 89.8383]	[0, 0.3937]	[21.4942, 61.7125]
t_{3N}	[5.0169, 68.6044]	[0, 0.9269]	[233.3634, 80.8133]
t_{4N}	[29.0354, 78.5532]	[0, 0.6304]	[40.3218, 70.5782]
t_{5N}	[3.8535, 56.0780]	[0, 0.9313]	[303.8204, 98.8650]
t_{6N}	[28.5793, 66.6949]	[0, 0.5715]	[40.9653, 83.1270]
t_{7N}	[4.7559, 68.4508]	[0, 0.9305]	[246.1690, 80.9947]
t_{8N}	[3.4460, 62.0306]	[0, 0.9444]	[339.7478, 89.3776]
t_{9N}	[3.8535, 56.0780]	[0, 0.9313]	[303.8204, 98.8650]
t^*_N	[3.2176, 52.7549]	[0, 0.9390]	[363.8591, 105.0925]

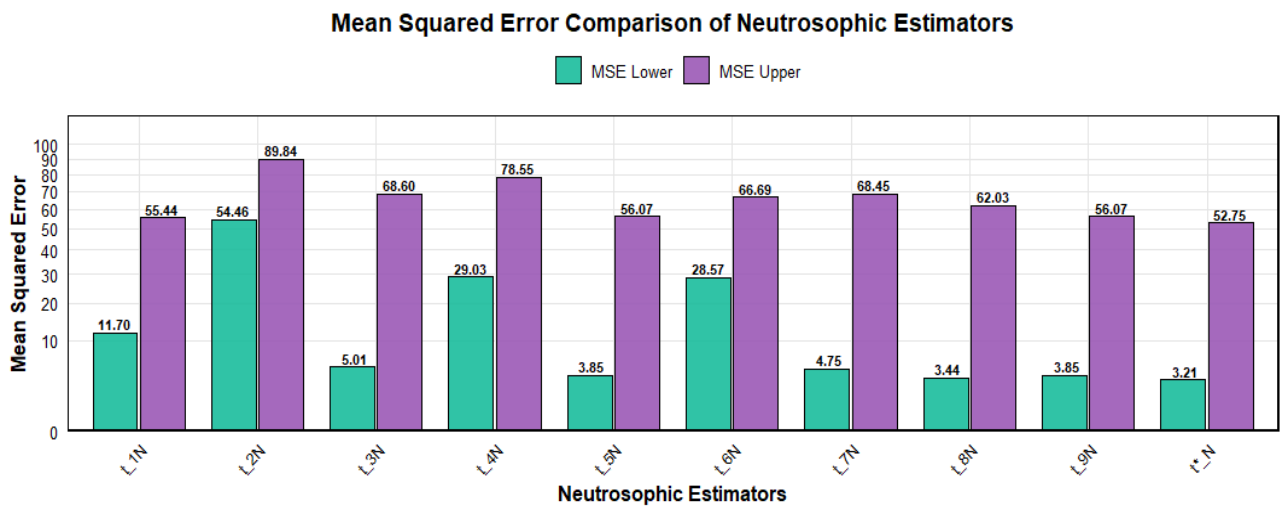


Figure 3. MSEs values for proposed estimator and various existing estimators for dataset 2.

7 | Simulation Study

In order to support the outcomes of the empirical results and verify the efficiency of our proposed estimator, a simulation study was undertaken on neutrosophic simulated data. For the simulation study, we adopted the approach proposed by [33] and [34]. The following steps were employed to generate neutrosophic data:

- The neutrosophic auxiliary variable X_N was simulated using a Neutrosophic Normal Distribution $X_N \sim NN(\mu_{xN}, \sigma_{xN}^2)$, This represents a case where the population mean μ_{xN} is indeterminate, and thus X_N lies within $X_N = [X_L, X_U]$; $\mu_{xN} \in [0.749, 1.165]$, $\sigma_{xN}^2 = [1, 3.204]$.
- To generate the neutrosophic study variable, we employed the model $Y_N = X_N - 7.2e_N$, where $e_N \sim NN(0, 1)$.
- Using neutrosophic simple random sampling without replacement, we selected three samples of sizes n_N ($n_1 = 100$, $n_2 = 150$ and $n_3 = 200$) from a population of size $N_N = 1000$.

Table 4. MSEs values of both traditional and newly suggested estimator for sample size $n = 100$ based on simulation analysis.

Estimator	MSE	I_N	PRE
t_{1N}	[0.4756, 0.4958]	[0, 0.0407]	[100, 100]
t_{2N}	[0.5033, 0.5835]	[0, 0.1374]	[94.49, 84.96]
t_{3N}	[0.4675, 0.4680]	[0, 0.0011]	[101.74, 105.94]
t_{4N}	[0.4704, 0.4765]	[0, 0.0128]	[101.10, 104.05]
t_{5N}	[0.4691, 0.4744]	[0, 0.0112]	[101.39, 104.51]
t_{6N}	[0.4870, 0.5322]	[0, 0.0849]	[97.66, 93.16]
t_{7N}	[0.4676, 0.4686]	[0, 0.0021]	[101.71, 105.79]
t_{8N}	[0.4755, 0.4951]	[0, 0.0396]	[100.03, 100.13]
t_{9N}	[0.4722, 0.4824]	[0, 0.0211]	[100.72, 102.77]
t_N^*	[0.2428, 0.3401]	[0, 0.2861]	[195.89, 145.76]

Table 5. MSEs values of both traditional and newly suggested estimator for sample size $n = 150$ based on simulation analysis.

Estimator	MSE	I_N	PRE
t_{1N}	[0.2994, 0.3116]	[0, 0.0392]	[100, 100]
t_{2N}	[0.3167, 0.3668]	[0, 0.1366]	[94.52, 84.95]
t_{3N}	[0.2942, 0.2942]	[0, 0.0000]	[101.74, 105.93]
t_{4N}	[0.2961, 0.2995]	[0, 0.0114]	[101.10, 104.05]
t_{5N}	[0.2953, 0.2982]	[0, 0.0097]	[101.39, 104.51]
t_{6N}	[0.3065, 0.3345]	[0, 0.0837]	[97.67, 93.16]
t_{7N}	[0.2943, 0.2946]	[0, 0.0010]	[101.72, 105.79]
t_{8N}	[0.2993, 0.3114]	[0, 0.0389]	[100.02, 100.09]
t_{9N}	[0.2972, 0.3032]	[0, 0.0198]	[100.72, 102.77]
t_N^*	[0.1820, 0.2371]	[0, 0.2324]	[164.51, 131.44]

Table 6. MSEs values of both traditional and newly suggested estimator for sample size $n = 200$ based on simulation analysis.

Estimator	MSE	I_N	PRE
t_{1N}	[0.2113, 0.2204]	[0, 0.0413]	[100, 100]
t_{2N}	[0.2237, 0.2593]	[0, 0.1373]	[94.47, 84.97]
t_{3N}	[0.2077, 0.2080]	[0, 0.0014]	[101.74, 105.95]
t_{4N}	[0.2090, 0.2118]	[0, 0.0132]	[101.10, 104.06]
t_{5N}	[0.2084, 0.2108]	[0, 0.0114]	[101.39, 104.51]
t_{6N}	[0.2164, 0.2365]	[0, 0.0850]	[97.65, 93.71]
t_{7N}	[0.2077, 0.2083]	[0, 0.0029]	[101.72, 105.81]
t_{8N}	[0.2113, 0.2202]	[0, 0.0404]	[100.01, 100.07]
t_{9N}	[0.2098, 0.2144]	[0, 0.0215]	[100.72, 102.77]
t_N^*	[0.1439, 0.1772]	[0, 0.1879]	[146.81, 124.36]

- The obtained samples were used to evaluate the MSEs of the existing as well as the proposed estimators.
- The entire procedure of obtaining MSEs for the estimators using neutrosophic simple random sampling was repeated **10,000** times.
- The outcomes of MSE and PRE evaluations for the estimators are summarized in Tables 4 to 6.

8 | Discussion and Results

According to Table 2, the sampling variance for the neutrosophic estimator t_{1N} lies in the interval [11.7951, 11.8082], while that of the neutrosophic exponential ratio estimator (t_{5N}) is [3.2531, 3.2569]. However, the range of the neutrosophic MSEs for estimator (t_{4N}) is [31.7638, 31.8282]. The reason estimator (t_{4N}) exhibits greater MSEs than estimator (t_{1N}) lies in the low value of neutrosophic CV associated with the auxiliary variable X. Additionally, Figure 2 displays the Mean Squared Error (MSE) values of the proposed estimator compared to existing estimators for Dataset 1. Among the various competing estimators for the neutrosophic population mean ($\bar{Y}_N$), the proposed class of neutrosophic estimators (t_N^*) has the lowest MSE, ranging from [2.6999, 2.7051], and demonstrate the higher PRE.

In a similar manner, Table 3 illustrates the comparison of MSEs and PREs between the proposed and existing estimators. Moreover, Figure 3 illustrates the MSE comparisons for Dataset 2, emphasizing the effectiveness of the proposed estimator over existing ones. Furthermore, the proposed estimators exhibit reduced MSEs and improved PREs when compared to those of the existing estimators. The bold style is used to highlight its value. Therefore, Tables 2 and 3 support the theoretical derivation that the proposed estimator is superior to the existing ones. Additionally, A simulation study based on sample sizes of 100, 150, and 200 is summarized in Tables 4-6, highlighting the comparative performance of the proposed and existing neutrosophic estimators with respect to MSEs and PREs. The findings reveal that the proposed estimator (t_N^*) is the most efficient among the estimators considered.

9 | Conclusion

This paper introduces a logarithmic ratio-type estimator that leverages auxiliary information to estimate the finite population mean when data is imprecise. A key feature of neutrosophic observations is their inclusion of imprecise, uncertain, or indeterminate values. We examined the bias and mean squared error (MSE) of the proposed estimator family within the framework of neutrosophic sampling, using a first-order approximation. We derived the neutrosophic optimum values of the estimator's characterizing scalars, the corresponding minimum MSE of the proposed estimator has been computed accordingly. Additionally, the proposed estimator was theoretically compared with competing neutrosophic estimators, and efficiency conditions were

derived. These efficiency conditions were verified using empirical analysis as well as through simulation study. Based on the empirical results presented in Tables 2 and 3, and the simulation findings in Tables 4 and 6, we conclude that the proposed estimators are effective in estimating the finite population mean using auxiliary information when the data is unclear, vague, ambiguous, indeterminate, or interval-based.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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