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On Strong Super Hyper EQ Algebras: A Proof-of-Principle Study

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Abstract

Today, in the study of algebra structures, we notice the new concept of superhyper algebras. In this article, we introduce and study strong $(\ell, \overline{q''})$ -superhyper *EQ* algebra. Also, we will examine the specific features of strong $(\ell, \overline{q''})$ -superhyper *EQ* algebra. The article aims to extend *EQ* algebras to super hyper *EQ* algebras using the q^{th} -power set of a set.

Keywords: *EQ* Algebra; Strong $(\ell, \overline{q''})$ -superhyper *EQ* Algebra; $(\ell, \overline{q''})$ -superhyper Operation.

1 | Introduction

Logical algebra is an interdisciplinary algebraic structure that is applicable in various sciences. In logical algebra, each set is uniquely defined by properties and algebraic principles and obeys a certain law. Considering the importance of the theory of logical algebras, many researchers investigate its characteristics and importance. Logical algebraic hyperstructures are more useful in the real world by covering the deficiencies of logical algebra, especially when dealing with relationships between sets of objects. For the first time, an interesting and important logical algebra denominated *EQ* algebra was Raised by Novak and De Baets in 2009 [12]. *EQ* algebras have three main binary operations and one top element which is assumed to be a commutative and associative multiplication. The logical background of *EQ* algebras is different from other logical algebras. Since the generalization of the residuated lattices are *EQ* algebras, therefore *EQ* algebras are interesting and important algebraic structures. Since residuated lattices generalizations are *EQ* algebras, *EQ* algebras are attractive and important algebraic structures [1-24]. We can read more about *EQ* algebras in [1,2,8,10-12].

The first time, Florentin Smarandache offered the concept of SuperHyperAlgebras as a generalization of Hyper Algebras, which, contrary to the limitation of Hyper Algebras, SuperHyperAlgebras is more applicable in the real world [16-20]. Recently, Rahmati and Hamidi introduced superhyper *G*-algebras as a generalization of *G*-algebras [14], also, Hamidi et al. introduced and investigated new concepts of superhyper algebras [4-6]. This paper is dedicated to the introduction and study of strong $(\ell, \overline{q''})$ -superhyper *EQ* algebras as a generalization and extension of *EQ* algebra. Also, some properties of strong $(\ell, \overline{q''})$ -superhyper *EQ* algebras



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have been investigated. Our intention in presenting this article is to offer a strong $(\ell, \overline{q''})$ -superhyper EQ algebras as an extension of logic algebras.

2 | Preliminaries

In this part, let's remind the preliminary notions.

Definition 1. [1] Suppose X be a non-absurd collection. Then a $(X, \widehat{\wedge}, \widehat{\otimes}, \widehat{*}, 1)$ where $\widehat{\wedge}, \widehat{\otimes}, \widehat{*}$ are binary operations, EQ algebra is called if for all $\hbar, \iota, b, \natural \in X$:

(EQ-1) $(X, \widehat{\wedge}, 1)$ is a $\widehat{\wedge}$ -semilattice which contains the element above 1. We put $\hbar \leq \iota$ if and only if $\widehat{\wedge}(\hbar, \iota) = \hbar$,

(EQ-2) $(X, \widehat{\otimes}, 1)$ is a commutative monoid and $\widehat{\otimes}$ is isotone,

(EQ-3) $\hbar \widehat{*} \hbar = 1$,

(EQ-4) $(\widehat{\wedge}(\hbar, \iota) \widehat{*} b) \widehat{\otimes} (\natural \widehat{*} \hbar) \leq (b \widehat{*} \widehat{\wedge}(\natural, \iota))$,

(EQ-5) $(\hbar \widehat{*} \iota) \widehat{\otimes} (b \widehat{*} \natural) \leq (\hbar \widehat{*} b) \widehat{*} (\iota \widehat{*} \natural)$,

(EQ-6) $\widehat{\wedge}(\hbar, \iota, b) \widehat{*} \hbar \leq \widehat{\wedge}(\hbar, \iota) \widehat{*} \hbar$,

(EQ-7) $\hbar \widehat{\otimes} \iota \leq \hbar \widehat{*} \iota$.

The action " $\widehat{\otimes}$ " is named multiplication, and " $\widehat{*}$ " is named fuzzy equality.

Definition 2. [17, 20] Suppose Y be a non-absurd collection. Then $(Y, \delta_{(r,s)}, 1)$ is named a (r, s) -super hyperalgebra, where $\delta_{(r,s)}: Y^r \rightarrow P_*^s(Y)$ is called an (r, s) -super hyper operation, $P_*^s(Y)$ is the s^{th} powerset of the collection Y , which does not include $\emptyset$, each $B \in P_*^s(Y)$, we know $\{B\}$ by B , $r \geq 2, s \geq 0, Y^r = \underbrace{Y \times Y \times \dots \times Y}_{r\text{-reps}}$. If $s = 0$, then $P_*^0(Y) = Y$.

3 | Superhyper EQ Algebra

In this part, we construct the notion of strong superhyper EQ algebras as generalizations of EQ algebras and present its specific features.

Definition 3. Presume Y be a non-absurd collection and $1 \in Y$. Then $(Y, \widehat{\wedge}, \widehat{\odot}, \delta_{(\ell, \overline{q''})}, 1)$ is named a strong $(\ell, \overline{q''})$ -superhyper EQ algebra, if for each $\hbar, \iota, b, \natural \in Y$:

- $(Y, \widehat{\wedge}, 1)$ is a $\widehat{\wedge}$ -semilattice which contains the element above 1. We put $\hbar \leq \iota$ iff $\widehat{\wedge}(\hbar, \iota) = \hbar$,
- $(Y, \widehat{\odot}, 1)$ is a commutative monoid and $\widehat{\odot}$ is isotone,
- $1 \in \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar)$,
- $\widehat{\odot}(\delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), b), \delta_{(\ell, \overline{q''})}(\natural, \natural, \dots, \natural, \hbar)) \leq \delta_{(\ell, \overline{q''})}(b, b, \dots, b, \widehat{\wedge}(\natural, \iota))$,
- $\widehat{\odot}(\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota), \delta_{(\ell, \overline{q''})}(b, b, \dots, b, \natural)) \leq \delta_{(\ell, \overline{q''})}(\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, b), \dots, \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, b), \delta_{(\ell, \overline{q''})}(\iota, \iota, \dots, \iota, \natural))$
- $\delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota, b), \dots, \widehat{\wedge}(\hbar, \iota, b), \hbar) \leq \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar)$
- $\widehat{\odot}(\hbar, \iota) \leq \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota)$.

Example 4. (i) Assume $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(\ell, \overline{q^n})}, 1)$ be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra. Then $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(2,0)}, 1)$ is an EQ algebra. (ii) Assume $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(\ell, \overline{q^n})}, 1)$ be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra. Then $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(2,1)}, 1)$ is a hyper EQ algebra.

Theorem 5. Assume $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(\ell, \overline{q^n})}, 1)$ be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra. Then $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(p,k)}, 1)$ is a strong (p, k) -superhyper EQ algebra, for each $k \geq q$.

Proof. Assume $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(\ell, \overline{q^n})}, 1)$ be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra and $k \geq q$. Because $P_*^q(Y) \subseteq P_*^k(Y)$, for each $\hbar_1, \hbar_2, \dots, \hbar_p \in X$, $\delta_{(\ell, \overline{q^n})}(\hbar_1, \hbar_2, \dots, \hbar_p) \subseteq \delta_{(p,k)}(\hbar_1, \hbar_2, \dots, \hbar_p)$. Therefore $1 \in \delta_{(\ell, \overline{q^n})}(\hbar_1, \hbar_2, \dots, \hbar_p)$ it means $1 \in \delta_{(p,k)}(\hbar_1, \hbar_2, \dots, \hbar_p)$. Therefore, all axioms are correct. $\square$

The coming precept is a result of other axioms of strong $(\ell, \overline{q^n})$ -superhyper EQ algebra:

$$\delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \hbar) \leq \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota, b), \dots, \widehat{\Lambda}(\hbar, \iota, b), \widehat{\Lambda}(\hbar, b)).$$

Definition 6. Assume Y be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra. We also for $\hbar, \iota \in Y$, set $\hbar \hookrightarrow \hbar \dots \hookrightarrow \hbar \hookrightarrow \iota := \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \hbar)$, $\hbar^* := \delta_{(\ell, \overline{q^n})}(\hbar, \dots, \hbar, 1)$. Therefore, we can rewrite (EQ_{sh} -6) and (1) as $\hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \widehat{\Lambda}(\iota, b) \leq \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota$, $\hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota \leq \widehat{\Lambda}(\hbar, b) \hookrightarrow \dots \hookrightarrow \widehat{\Lambda}(\hbar, b) \hookrightarrow \iota$, respectively. If Y also contains a bottom element 0 therefore, we can define the following unary operation $\neg$ on Y with $\neg \hbar := \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, 0)$ and call $\neg \hbar$ a negation of $\hbar \in Y$.

Theorem 7. Presume $(Y, \widehat{\Lambda}, \widehat{\odot}, \delta_{(\ell, \overline{q^n})}, 1)$ be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra. Then, the coming features are available for every $\hbar, \iota, b \in Y$:

- $\widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota), \delta_{(\ell, \overline{q^n})}(\iota, \iota, \dots, \iota, b)) \leq \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hbar),$
- $\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar) \leq \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \widehat{\Lambda}(\hbar, \iota)),$
- $\widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar), \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), b)) \leq \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \widehat{\Lambda}(\hbar, \iota)),$
- $\delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \hbar) \leq \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, b), \dots, \widehat{\Lambda}(\hbar, b), \widehat{\Lambda}(\widehat{\Lambda}(\hbar, \iota), b)).$

Proof. By (EQ_{sh} -4), we have

$$\begin{aligned} & \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota), \delta_{(\ell, \overline{q^n})}(\iota, \iota, \dots, \iota, b)) \\ &= \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\iota, \iota, \dots, \iota, b), \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota)) \\ &= \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\iota, 1), \dots, \widehat{\Lambda}(\iota, 1), b), \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota)) \leq \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \widehat{\Lambda}(\hbar, 1)) \\ &= \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hbar). \end{aligned}$$

By (EQ_{sh} -4), we get

$$\begin{aligned} & \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar) \\ &= \widehat{\odot}(1, \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar)) \\ &\leq \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar)) \leq \\ & \delta_{(\ell, \overline{q^n})}(\widehat{\Lambda}(\hbar, \iota), \dots, \widehat{\Lambda}(\hbar, \iota), \widehat{\Lambda}(\hbar, \iota)). \end{aligned}$$

According to (i), (ii) and (EQ_{sh-2}) , we have

$$\begin{aligned} & \widehat{\odot} \left(\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota), \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), b) \right) \\ & \leq \widehat{\odot} \left(\delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \widehat{\wedge}(\hbar, \iota)), \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), b) \right) \\ & \leq \delta_{(\ell, \overline{q''})}(b, b, \dots, b, \widehat{\wedge}(\hbar, \iota)). \end{aligned}$$

By (EQ_{sh-3}) , (EQ_{sh-4}) , we have

$$\begin{aligned} & \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar) \\ & \subseteq \widehat{\odot} \left(\delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, b), \dots, \widehat{\wedge}(\hbar, b)), \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar) \right) \\ & \leq \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, b), \dots, \widehat{\wedge}(\hbar, b), \widehat{\wedge}(\widehat{\wedge}(\hbar, \iota), b)). \end{aligned}$$

Definition 8. Assume Y be a strong $(\ell, \overline{q''})$ -superhyper EQ algebra. Then Y is named

- semi-separated if for every $\hbar \in Y$, $\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, 1) = 1 \Rightarrow \hbar = 1$.
- separated if for every $\hbar, \iota \in Y$, $\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota) = 1 \Rightarrow \hbar = \iota$.
- spanned If 0 is the bottom element of Y and $\delta_{(\ell, \overline{q''})}(0, 0, \dots, 0) = 0$.
- good if for every $\hbar \in Y$, $\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, 1) = \hbar$.
- residuated if for every $\hbar, \iota, b \in Y$, $\widehat{\wedge}(\widehat{\odot}(\hbar, \iota), b) = \widehat{\odot}(\hbar, \iota) \Leftrightarrow \widehat{\wedge}(\hbar, \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\iota, b), \dots, \widehat{\wedge}(\iota, b), \iota)) = \hbar$.
- idempotent it satisfies $\widehat{\odot}(\hbar, \hbar, \dots, \hbar) = 1$.
- involutive (strong $(\ell, \overline{q''})$ -superhyper IEQ -algebra) if for every $\hbar \in Y$, $\neg\neg\hbar = \hbar$.
- a lattice-strong $(\ell, \overline{q''})$ -super hyper EQ algebra (a strong $(\ell, \overline{q''})$ -superhyper ℓEQ algebra) if it is a lattice-ordered strong $(\ell, \overline{q''})$ -super hyper EQ algebra where in the coming precept holds for each $\hbar, \iota, b, \iota \in Y$, $\widehat{\odot}(\delta_{(\ell, \overline{q''})}(\vee(\hbar, \iota), \dots, \vee(\hbar, \iota), b), \delta_{(\ell, \overline{q''})}(\iota, \iota, \dots, \iota, \hbar)) \leq \delta_{(\ell, \overline{q''})}(\vee(\hbar, \iota), \dots, (\vee(\hbar, \iota), b))$.

Theorem 9. All strong $(\ell, \overline{q''})$ -super hyper EQ -algebras have the coming properties for each $\hbar, \iota, b \in Y$:

- $\widehat{\odot}(\hbar, \iota) \leq \widehat{\wedge}(\hbar, \iota) \leq \hbar, \iota$ and $\widehat{\odot}(\iota, \hbar) \leq \widehat{\wedge}(\hbar, \iota) \leq \hbar, \iota$,
- $\hbar = \iota$ implies $1 \in \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, b)$,
- $\iota \leq \iota^*$,
- $\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota) \leq \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota$ and $1 \in \hbar \hookrightarrow \dots \hookrightarrow \hbar$,
- Presume $\hbar \leq \iota$. Then,

$$\begin{aligned} & 1 \in \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota, \\ & \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota) = \iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hbar, \\ & \hbar^* \leq \iota^*, \\ & b \hookrightarrow \dots \hookrightarrow b \hookrightarrow \hbar \leq b \hookrightarrow \dots \hookrightarrow b \hookrightarrow \iota \text{ and } \iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow b \leq \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow b. \end{aligned}$$

Proof. According to the properties of isotonic and monoid $\widehat{\odot}$, $\hbar \leq \hbar$ and $\iota \leq 1$, so $\widehat{\odot}(\hbar, \iota) \leq \widehat{\odot}(\hbar, 1) = \hbar$. Also $\hbar \leq 1$ and $\iota \leq \iota$, so $\widehat{\odot}(\hbar, \iota) \leq \widehat{\odot}(1, \iota) = \iota$. Therefore $\widehat{\odot}(\hbar, \iota) \leq \widehat{\wedge}(\hbar, \iota)$. Other $\widehat{\wedge}(\hbar, \iota) \leq \hbar$ and $\widehat{\wedge}(\hbar, \iota) \leq \iota$. So $\widehat{\odot}(\hbar, \iota) \leq \widehat{\wedge}(\hbar, \iota) \leq \hbar, \iota$.

If $\hbar = \iota$, then $\delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, b) = \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar)$. According to (EQ_{sh-3}) , the sentence is proved.

According to the (EQ_{sh-3}) , $\iota = \widehat{\odot}(\iota, 1) \leq \delta_{(\ell, \overline{q''})}(\iota, \iota, \dots, \iota, 1) = \iota^*$.

By Theorem 7(i),

$$\begin{aligned} & \delta_{(\ell, \overline{q''})}(\hbar, \hbar, \dots, \hbar, \iota) \leq \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\iota, \hbar), \dots, \widehat{\wedge}(\iota, \hbar), \widehat{\wedge}(\hbar, \hbar)) \\ & = \delta_{(\ell, \overline{q''})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar) = \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota. \end{aligned}$$

As well as

$$\begin{aligned}\hbar \hookrightarrow \dots \hookrightarrow \hbar &= \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\hbar, \hbar), \dots, \widehat{\wedge}(\hbar, \hbar), \hbar) \\ &= \delta_{(\ell, \overline{q^n})}(\hbar, \dots, \hbar).\end{aligned}$$

Since $1 \in \delta_{(\ell, \overline{q^n})}(\hbar, \dots, \hbar)$, $1 \in \hbar \hookrightarrow \dots \hookrightarrow \hbar$.

Suppose $\hbar \leq \iota$, then $\widehat{\wedge}(\hbar, \iota) = \hbar$ and

$$\begin{aligned}\hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar) &= \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar), \\ 1 \in \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar),\end{aligned}$$

Therefore $1 \in \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow \iota$. Also

$$\begin{aligned}\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hbar \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), \hbar) \\ = \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \hbar).\end{aligned}$$

Considering (EQ_{sh-6}) ,

$$\begin{aligned}\hbar^* &= \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, 1) \\ &= \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\hbar, \iota), \dots, \widehat{\wedge}(\hbar, \iota), 1) \\ &= \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\iota, \hbar), \dots, \widehat{\wedge}(\iota, \hbar), 1) \\ &= \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(1, \iota, \hbar), \dots, \widehat{\wedge}(1, \iota, \hbar), 1) \\ &\leq \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(1, \iota), \dots, \widehat{\wedge}(1, \iota), 1) \\ &= \delta_{(\ell, \overline{q^n})}(\iota, \iota, \dots, \iota, 1) = \iota^*.\end{aligned}$$

By (EQ_{sh-6}) ,

$$\begin{aligned}b \hookrightarrow \dots \hookrightarrow b \hookrightarrow \hbar \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \hbar), \dots, \widehat{\wedge}(b, \hbar), b) \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \hbar, \iota), \dots, \widehat{\wedge}(b, \hbar, \iota), b) \leq \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \iota), \dots, \widehat{\wedge}(b, \iota), b) = b \hookrightarrow \dots \hookrightarrow b \hookrightarrow \iota.\end{aligned}$$

By Theorem 7(iv),

$$\begin{aligned}\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow b \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \iota), \dots, \widehat{\wedge}(b, \iota), \iota) \\ \leq \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \iota, \hbar), \dots, \widehat{\wedge}(b, \iota, \hbar), \widehat{\wedge}(\iota, \hbar)) \\ = \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(b, \hbar), \dots, \widehat{\wedge}(b, \hbar), \hbar) \\ = \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow b.\end{aligned}$$

Lemma 10. Assume $\hbar \leq \iota \leq b$. Then $\delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \iota) \leq \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hbar)$ and $\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, b) \leq \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota)$.

Proof. By Theorem 9(e), assume $\hbar \leq \iota$ then $\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow b \leq \hbar \hookrightarrow \dots \hookrightarrow \hbar \hookrightarrow b$, hence $\delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \iota) \leq \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hbar)$. As well as assume $\iota \leq b$ then $b \hookrightarrow \dots \hookrightarrow b \hookrightarrow \hbar \leq \iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hbar$, therefore $\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, b) \leq \delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota)$.

Theorem 11. All strong $(\ell, \overline{q^n})$ -super hyper EQ -algebras have the following properties:

- $\widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hbar, \hbar, \dots, \hbar, \iota), \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hbar)) \leq \delta_{(\ell, \overline{q^n})}(\widehat{\wedge}(\hbar, b), \dots, \widehat{\wedge}(\hbar, b), \widehat{\wedge}(\iota, \hbar)),$

- $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \hat{h}) \leq$
 $\delta_{(\ell, \overline{q^n})}(\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b), \dots, \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b),$
 $\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b)),$
- $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \hat{h}) \leq$
 $\delta_{(\ell, \overline{q^n})}(\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, b), \dots, \delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, b), \delta_{(p, q)}(\hat{h}, \hat{h}, \dots, \hat{h}, b)),$
- $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \hat{h}) \leq$
 $\delta_{(\ell, \overline{q^n})}((\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}), \dots, (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}), (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h})),$
- $\hat{h} \hookrightarrow \dots \hookrightarrow \hat{h} \hookrightarrow \hat{h} \leq$
 $((\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}) \hookrightarrow \dots \hookrightarrow (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}) \hookrightarrow (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h})).$

Proof. By Theorem 7 (ii), $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \iota) \leq \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, b), \dots, \hat{\wedge}(\iota, b), \hat{\wedge}(\hat{h}, b))$ and $\delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hat{h}) \leq \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, \hat{h}), \dots, \hat{\wedge}(\iota, \hat{h}), \hat{\wedge}(\iota, b))$.

Therefore, according to the characteristics of $\widehat{\odot}$ and Theorem 7 (a), we get

$$\begin{aligned} & \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \iota), \delta_{(\ell, \overline{q^n})}(b, b, \dots, b, \hat{h})) \\ & \leq \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, b), \dots, \hat{\wedge}(\iota, b), \hat{\wedge}(\hat{h}, b)), \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, \hat{h}), \dots, \hat{\wedge}(\iota, \hat{h}), \hat{\wedge}(\iota, b))) \\ & = \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, \hat{h}), \dots, \hat{\wedge}(\iota, \hat{h}), \hat{\wedge}(\iota, b)), \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\iota, b), \dots, \hat{\wedge}(\iota, b), \hat{\wedge}(\hat{h}, b))) \\ & \leq \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, b), \dots, \hat{\wedge}(\hat{h}, b), \hat{\wedge}(\iota, \hat{h})). \end{aligned}$$

By Theorem 7 (b) and (EQ_{sh}-5), we conclude

$$\begin{aligned} & \delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \hat{h}) \leq \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), \hat{\wedge}(\hat{h}, \iota)) \\ & \leq \widehat{\odot}(\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), \hat{\wedge}(\hat{h}, \iota)), \delta_{(\ell, \overline{q^n})}(b, b, \dots, b)) \\ & \leq \delta_{(\ell, \overline{q^n})}(\delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b), \dots, \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b), \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \iota), \dots, \hat{\wedge}(\hat{h}, \iota), b)). \end{aligned}$$

By putting $\iota = 1$ in (b) is obtained.

By putting $b = b$ in (b) and according to the Definition 6 (2) is obtained.

Using (d), we achieve

$$\begin{aligned} & \hat{h} \hookrightarrow \dots \hookrightarrow \hat{h} \hookrightarrow \hat{h} \\ & = \delta_{(\ell, \overline{q^n})}(\hat{\wedge}(\hat{h}, \hat{h}), \dots, \hat{\wedge}(\hat{h}, \hat{h}), \hat{h}) \\ & \leq \delta_{(\ell, \overline{q^n})}((\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}, \dots, \iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}), (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{\wedge}(\hat{h}, \hat{h}))) \\ & \leq (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}, \dots, \iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}) \hookrightarrow (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{\wedge}(\hat{h}, \hat{h})) \\ & \leq ((\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}) \hookrightarrow \dots \hookrightarrow (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h}) \hookrightarrow (\iota \hookrightarrow \dots \hookrightarrow \iota \hookrightarrow \hat{h})). \end{aligned}$$

(by Definition 6 (4)).

The next result is directly obtained from Theorem 11.

Corollary 12. Assume Y be a strong $(\ell, \overline{q^n})$ -superhyper EQ algebra with bottom element 0 . Then for each $\hat{h}, \iota, b \in Y$:

- $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \iota) \leq \delta_{(\ell, \overline{q^n})}(\neg \hat{h}, \neg \hat{h}, \dots, \neg \hat{h}, \neg \iota)$. Moreover, if Y is involutive, thus $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, \iota) = \delta_{(\ell, \overline{q^n})}(\neg \hat{h}, \neg \hat{h}, \dots, \neg \hat{h}, \neg \iota)$.
- $\delta_{(\ell, \overline{q^n})}(\hat{h}, \hat{h}, \dots, \hat{h}, b) \leq \delta_{(\ell, \overline{q^n})}(\neg \hat{\wedge}(\hat{h}, \iota), \dots, \neg \hat{\wedge}(\hat{h}, \iota), \neg \hat{\wedge}(b, \iota))$.

4 | Conclusion

This article is dedicated to introducing the new concept of strong superhyper *EQ* algebras as an expansion of *EQ* algebras. We hope that this research will be used in future studies in the field of logical superalgebras. We also hope that more valuable research will be done on Neutrosophic unique-valued (super) algebras *EQ* and the use of these algebras in various fields of uncertainty and fuzzy math, where this method is more powerful than classical mathematics.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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