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The Seasonal Factors Analysis of the Neutrosophic Time Series (Neutrosophic Seasonal Index)

Rafif Alhabib ^{1,*}  and Bushra Alsaeed ² 

¹ Department of Mathematics, Faculty of Science, Hama University, Hama, Syria; rafif.alhabib85@gmail.com.

² Faculty of Science, Aleppo University, Syria; alsaeedbushra41@gmail.com.

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Abstract

The paper studies and analyzes the seasonal components of a neutrosophic time series. The methodology follows a structure similar to that of classical logic but differs in the treatment of data. In this context, all data—whether precisely defined or indeterminate are considered. Each time unit (t) may correspond to a set or range of values instead of a single value (As an example of indeterminate values). This approach allows for a more precise description of the time series behavior and enables more accurate forecasting than the classical method. The study aims to identify the seasonal impact on the neutrosophic time series and determine how it can be excluded or predicted, by computing the Neutrosophic Seasonal Index (S_N).

Keywords: Time Series; Neutrosophic Logic; Neutrosophic Time Series; Seasonal Components.

1 | Introduction

Neutrosophic is a novel modeling approach designed to effectively handle the inherent uncertainties of the real world. It replaces binary logic, which recognizes only true or false, by introducing a third neutral state interpreted as indeterminate or uncertain. The American philosopher and mathematician Florentin Smarandache founded this logic in 1999 as a generalization of fuzzy logic and an extension of fuzzy set theory proposed by Lotfi A. Zadeh in 1965. Neutrosophic logic has grown significantly in recent years, with applications in measurement, set theory, graphs, and various scientific and practical fields [4, 7-13].

The emergence of this logic stems from our limited and relative understanding of reality, necessitating a logical system that accommodates incomplete data and enables reasoning both in everyday life and in scientific practice in all its forms.

Smarandache defined neutrosophic logic [1] as a new non-classical logic that studies the origin, nature, and domain of indeterminacy, as well as the interaction of all spectrums imaginable by humans regarding a certain issue. It takes into account every idea along with its opposite and the spectrum of indeterminacy.

The main idea of neutrosophic logic is to characterize each logical statement in three dimensions: the degree of truth (T), the degree of indeterminacy (I), and the degree of falsehood (F), expressed as (T, I, F) [3]. These dimensions are incorporated into the analysis, providing a more precise description of the data and reducing randomness, which contributes to achieving high-precision results that support optimal decision-making.



Corresponding Author: rafif.alhabib85@gmail.com



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Classical logic studies a case along with its negation but ignores the state of indeterminacy, which is an explicit quantity and a core component in neutrosophic logic [2, 6]. This provides a more accurate description of the study and leads to more reliable results.

This research focuses on studying and analyzing the seasonal components of a time series within the framework of neutrosophic logic. In previous work [14], we defined the concept of the neutrosophic time series and introduced both linear and logarithmic neutrosophic models, including examples demonstrating how to deal with indeterminate values in time series data. We also tested the significance of the model parameters using the Student's *t*-distribution. Additionally, we presented the use of moving averages to pave neutrosophic time series [5].

In this study, we propose a method to analyze the seasonal effects in neutrosophic time series. This is done using a methodology similar to that of classical logic, with the difference that the analyzed time series is neutrosophic, as opposed to being classical.

1.1 | Research Importance and Objective

The importance of this study lies in its objectives, as follows:

- The significance of this research lies in its objective: to develop and generalize mathematical models used in the analysis of time series so that they encompass all variables and observations of the series without excluding any data.
- The main goal of analyzing time series within the neutrosophic logic framework is to understand the behavior of studied variables over time—including those that are uncertain or undefined in an effort to establish a dependable pattern. This pattern can then be used to predict the future behavior of these variables, under the assumption that past events influence future outcomes.

2 | Discussion and Methodology

Neutrosophic Time Series [14]:

A time series is a group of data points ordered in time. These data points are typically dependent on one another, which allows for reliable forecasting. It is also defined as a group of successive values (observations) that describe the development of a certain phenomenon over time. A time series is considered neutrosophic if some or all of its values, (observations) are indeterminate—meaning they may appear as a group or a range of values rather than a single exact number. That is, if some or all of the successive observations that describe the evolution of a certain phenomenon over time are imprecisely or indeterminately defined, the time series is classified as neutrosophic.

Components of a Time Series:

A time series in statistics consists of four specific and defined components, which together constitute the complete structure of a statistical time series, namely:

- **Secular Trend:** This refers to the long-term direction of the data. If the series shows an upward curve, it has an increasing trend; if downward, a decreasing trend. Causes include technological advancement and population growth. Examples include the rising sales of mobile devices and the declining sales of CDs.
- **Seasonal Variations:** These represent short-term, recurring movements that occur in the same direction and at the same times within a year. Examples include increased sales of refrigerators during summer, heaters during winter, and clothing during holidays. These seasonal patterns are typically caused by factors such as climate and social or cultural traditions.

- **Cyclical Variations:** When the curve of the annual time series takes the form of successive waves over a long period, the series is then influenced by cyclical factors such as economic cycles, climatic cycles, or population growth in a certain region.
- **Irregular Variations:** They represent sudden movements in the time series curve, either upward or downward, in an irregular manner. The causes of these variations include natural disasters, wars, and crises.

All four components are influenced by external factors, whether environmental, political, economic, social, or others.

Focus of This Research:

In this research, we focus on studying and analyzing the seasonal components of neutrosophic time series:

As introduced above, we understand the nature of seasonal variations and their causes. We now aim to assess the impact of seasonality on the behavior of the neutrosophic time series and explore how it can be isolated or predicted. This is done by computing the Neutrosophic Seasonal Index (S_N).

Computation of the Neutrosophic Seasonal Index (S_N): The index can be computed under two main scenarios:

a) When the time series is not affected by a general trend:

If the time series is not affected by a general trend, the neutrosophic seasonal index is calculated using the ratio-to-overall-mean method.

We will illustrate this through the following example Table 1, which represents the quarterly sales of a store over five years.

Table 1. The quarterly sales of a store over five years.

Sales/Year	Q1	Q2	Q3	Q4	Total
2020	10	[15 , 20]	30	[45 , 50]	[100 , 110]
2021	[15 , 17]	25	[32 , 35]	45	[117 , 122]
2022	[9 ,10]	[15 ,20]	30	[41 , 45]	[95 , 105]
2023	20	[25 , 30]	[32 ,40]	50	[127 , 140]
2024	15	25	[30 , 35]	50	[120 , 125]
Total	[69 , 72]	[105 , 120]	[154 , 170]	[231 , 240]	[559 , 602]
Quarterly Average	[13.8 , 14.4]	[21 , 24]	[30.8 , 34]	[46.2 , 48]	
Neutrosophic Seasonal Index (S_N)	[45.8 , 51.5]	[69.8 ,85.9]	[102.3 ,121.6]	[153.5, 171.7]	[371.4 , 430.7]

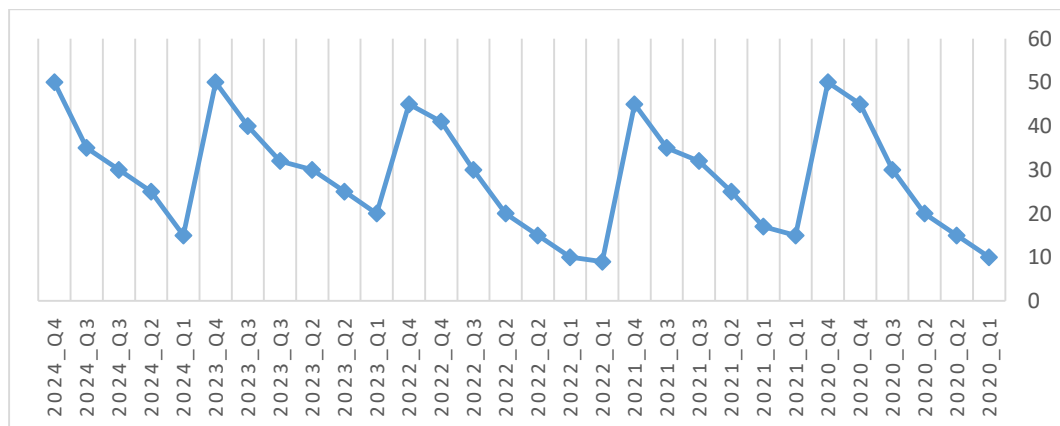


Figure 1. The quarterly sales of a store over five years.

"First, the average for each quarter is obtained by dividing the total of each quarter by the number of years (5). Then, the overall average was calculated by dividing the total [559, 602] by the total number of data points (20):

$$\frac{[559 . 602]}{20} = [27.95 . 30.1]$$

Then, the Neutrosophic Seasonal Index (S_N) is calculated:

$$S_N = \frac{\text{Quarterly Average}}{\text{Overall Average}} * 100$$

We find that:

$$S_{N1} = \frac{[13.8 . 14.4]}{[27.95 . 30.1]} * 100 = [45.8 . 51.5]$$

$$S_{N2} = \frac{[21 . 24]}{[27.95 . 30.1]} * 100 = [69.8 . 85.9]$$

$$S_{N3} = \frac{[30.8 . 34]}{[27.95 . 30.1]} * 100 = [102.3 . 121.6]$$

$$S_{N4} = \frac{[46.2 . 48]}{[27.95 . 30.1]} * 100 = [153.5 . 171.7]$$

$$S_{N1} + S_{N2} + S_{N3} + S_{N4} = [371.4, 430.7]$$

- It is observed that the data is not affected by the general trend, as indicated by the annual total.
- We find that the number of seasons multiplied by 100 belongs to the range of the sum of the neutrosophic seasonal indices. In contrast, under classical logic, the sum of seasonal indices results in a single fixed value equal to the number of seasons multiplied by 100.
- However, when working with neutrosophic logic which takes into account all time series values, whether precisely defined or indeterminate we obtain ranges of values instead of single absolute numbers.

b) The Neutrosophic Seasonal Index when the seasonal series is affected by a general trend:

To compute the Neutrosophic Seasonal Index, the general trend equation is first determined, followed by the calculation of the trend values. The Neutrosophic Seasonal Index is then calculated based on these values, as illustrated in the following example:

The following Table 2 presents the quarterly sugar sales (in tons) for a commercial store.

Table 2. The quarterly sugar sales (in tons) for a commercial store.

Sales/ Year	Q1	Q2	Q3	Q4	Total
2020	[10, 12]	20	30	40	[100,102]
2021	20	30	[45,50]	50	[145,150]
2022	58	40	50	[60, 65]	[208,213]
2023	40	[50,52]	60	70	[220,222]
2024	[60, 62]	70	80	90	[300,302]
Total	[188, 192]	[210, 212]	[265, 270]	[310, 315]	[973,989]

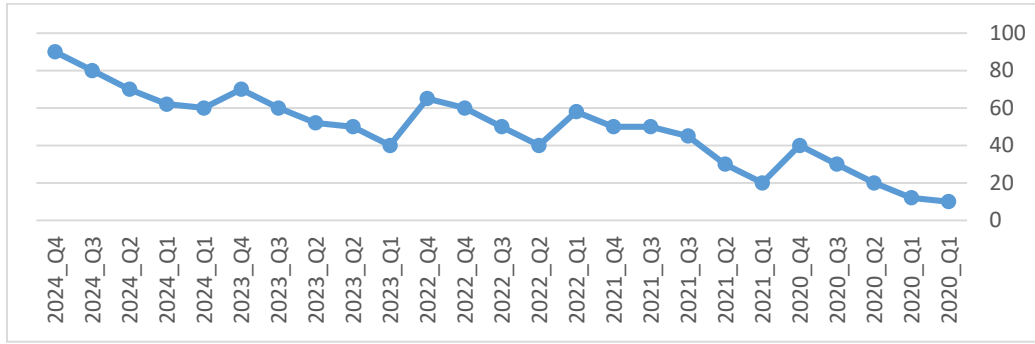


Figure 2. The quarterly sugar sales (in tons) for a commercial store.

We now calculate the constants of the neutrosophic seasonal trend equation.

$$\bar{Y} = [48.6, 49.5]$$

$$\bar{T} = \frac{210}{20} = 10.5$$

$$\sum t = 210$$

$$\sum t^2 = 2870$$

$$\sum y = [973, 989]$$

$$\sum yt = [12327, 12495]$$

$$b_N = \frac{\sum yt - \bar{t} \sum y}{\sum t^2 - \bar{t} \sum t}$$

$$= \frac{[12327, 12495] - 10.5 * [973, 989]}{2870 - 10.5 * (210)}$$

$$= \frac{[2110.5, 2110.5]}{665}$$

$$= 3.17$$

$$a_N = \bar{y} - b \bar{t}$$

$$= [48.6, 49.5] - 3.17 (10.5)$$

$$= [48.6, 49.5] - 33.285$$

$$= [15.3, 16.2]$$

Thus, the neutrosophic quarterly trend equation is:

$$\hat{y}_t = [15.3, 16.2] + 3.17 t$$

Next, we use this equation to compute the trend values, which represent the sales influenced only by the general trend. This yields the following Table 3:

Table 3. The sales are influenced only by the general trend.

Sales/Year	Q1	Q2	Q3	Q4
2020	[18.47 , 19.37]	[21.6 , 22.5]	[24.8 , 25.71]	[27.9 , 28.9]
2021	[31.15 , 32.05]	[34.32 , 35.2]	[37.5 , 38.4]	[40.7 , 41.6]
2022	[43.8 , 44.7]	[47 , 47.9]	[50.2 , 51.1]	[53.3 , 54.2]
2023	[56.51 , 57.4]	[59.7 , 60.6]	[62.9 , 63.8]	[66.02 , 66.9]
2024	[69.2 , 70.09]	[72.4 , 73.3]	[75.5 , 76.4]	[78.7 , 79.6]
Total	[219.13 , 223.6]	[235.05 , 239.5]	[250.9 , 255.4]	[266.6 , 271.2]

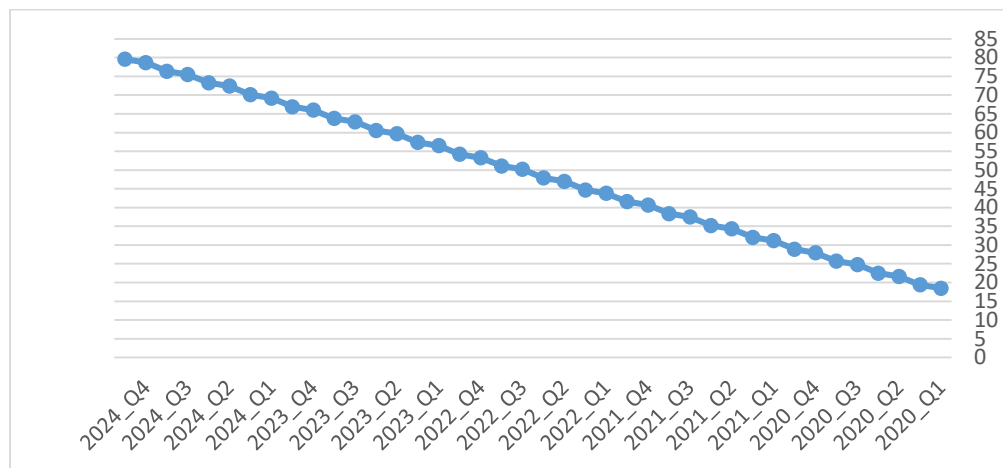


Figure 3. The sales are influenced only by the general trend.

We now compute the ratio of the actual values to the trend values for all data points.

$$S_n = \frac{y_t}{T} * 100$$

Thus, we obtain the following Table 4.

Table 4. Raw neutrosophic seasonal index and adjusted.

Sales/Year	Q1	Q2	Q3	Q4	Total
2020	[51.6,64.97]	[88.9,92.6]	[116.7,120.9]	[138.4,143.4]	
2021	[62.4,64.2]	[85.2,87.4]	[117.2,133.3]	[120.2,122.8]	
2022	[129.7,132.4]	[83.5,85.1]	[97.8,99.6]	[110.7,121.95]	
2023	[69.7,70.8]	[82.5,87.1]	[94.04,95.4]	[104.6,106.03]	
2024	[85.6,89.6]	[95.5,96.7]	[104.7,105.96]	[113.1,114.4]	
TOTAL	[339,421.97]	[435.6,448.9]	[530.44,555.16]	[587,608.6]	
Raw Neutrosophic Seasonal Index	[79.8,84.4]	[87.12,89.78]	[106.1,111.03]	[117.4,121.72]	[390.42,406.93]
Adjusted Neutrosophic Seasonal Index	[78.4,86.47]	[85.64,91.98]	[104.29,113.75]	[115.40,24.71]	[383.73,416.91]

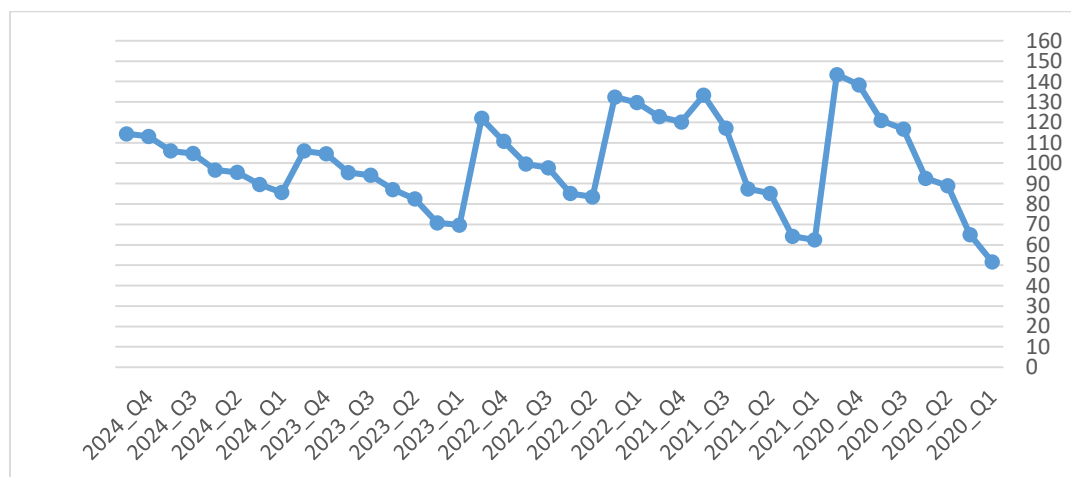


Figure 4. Neutrosophic seasonal variations

The raw neutrosophic seasonal index is the arithmetic mean of the values in each column.

The adjusted neutrosophic seasonal index is obtained by correcting the raw neutrosophic seasonal index through multiplication by the correction factor, which is given by the formula:

$$S_N = \frac{S_N \text{ raw}}{\text{Observed Total}} * \text{Theoretical Sum}$$

Where the Observed Total is: [390.42, 406.93]

The theoretical total is the number of seasons multiplied by 100. In our example, it is (400).

$$S_{N1} = \frac{[79.8, 84.4]}{[390.42, 406.93]} * 400 = [78.4, 86.47]$$

$$S_{N2} = \frac{[87.12, 89.78]}{[390.42, 406.93]} * 400 = [85.64, 91.98]$$

$$S_{N3} = \frac{[106.1, 111.03]}{[390.42, 406.93]} * 400 = [104.29, 113.75]$$

$$S_{N4} = \frac{[117.4, 121.72]}{[390.42, 406.93]} * 400 = [115.40, 124.71]$$

Meaning and Applications of the Seasonal Index:

- If $S_N = 100 \rightarrow$ There is no seasonal effect.
- If $S_N > 100 \rightarrow$ There is a positive seasonal effect.
- If $S_N < 100 \rightarrow$ There is a negative seasonal effect.

Uses of the Neutrosophic Seasonal Index (S_N):

- It is used to eliminate the seasonal effect from the neutrosophic time series meaning to obtain the values of the phenomenon without seasonal influence. This is done by dividing by the Neutrosophic Seasonal Index (S_N).
- It is also used to incorporate the seasonal effect into the neutrosophic time series, meaning to obtain the values of the phenomenon with the seasonal influence. This is done by multiplying by the Neutrosophic Seasonal Index (S_N).

Returning to our example:

- 1) The forecasted sales for the first quarter of 2025, affected only by the general trend in this quarter are $t = 21$, therefore the expected sales are:

$$\begin{aligned} \hat{y}_t &= [15.3, 16.2] + 3.17 t \\ &= [15.3, 16.2] + 3.17 (21) \\ &= [81.87, 82.77] \text{ tons} \end{aligned}$$

- 2) Forecasted sales for the first quarter of 2025, influenced by both the general trend and the seasonal effect, are:

To account for the seasonal effect, the trend value multiplied by the neutrosophic seasonal index for the first quarter. [78.4, 86.47]%

$$[78.4, 86.47]\% * [81.87, 82.77] = [64.18, 71.57]$$

- 3) The forecasted sales for the third quarter of 2024 assuming no seasonal effect are:

Here, we divide the actual value by the neutrosophic seasonal index for the third quarter.

That is, the forecasted sales assuming no seasonal effect are: [104.29, 113.75]

$$\frac{[80, 80]}{[104.29, 113.75]\%} = [70.3, 76.7] \text{ tons}$$

3 | Conclusion

- We conclude that dealing with neutrosophic time series provides a comprehensive and inclusive analysis of the studied case, without excluding any data merely because it is not explicitly defined.
- We observed that the presence of indeterminacy in the problem affects the estimated values. Therefore, indeterminate values cannot be ignored or excluded from the scope of the study in order to obtain results that are as accurate as possible, and thus better forecast and understand the future of the series.
- Classical logic is no longer sufficient to handle all types of data encountered in current analytical contexts. Scientific advancement has brought forth many new problems that cannot be interpreted within the framework of classical theory. At this point, neutrosophic logic plays a significant role, as it generalizes both classical and fuzzy logic, offering a more inclusive framework for interpreting and expanding the scope of the data under study.
- We conclude that presenting the seasonal component of the time series within the framework of neutrosophic logic provides a more general and clearer perspective in cases of uncertainty. It opens the way for a more flexible handling of the data encountered in the study and leads to better results in terms of optimal and more precise identification of the range of seasonal changes observed in the series. This contributes to a more accurate analysis of the time series, allowing for a more precise description of its behavior and a more accurate prediction of its future compared to the classical case.
 - We look forward in the near future to conducting an in-depth study of the remaining components of the time series within the framework of neutrosophic logic.
 - We recommend relying on neutrosophic time series when studying the evolution of any phenomenon over time, as they provide a more accurate and comprehensive future-oriented perspective.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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