

Paper Type: Original Article

Plithogenic Line Graph, Star Graph, and Regular Graph

Takaaki Fujita^{1*} 

¹ Independent Researcher, Tokyo, Japan; Takaaki.fujita060@gmail.com.

Received: 14 Jan 2025

Revised: 29 Mar 2025

Accepted: 16 Aug 2025

Published: 20 Aug 2025

Abstract

Graph theory, a branch of mathematics, studies relationships between entities using vertices and edges. In this field, Uncertain Graph Theory has emerged to capture the uncertainties inherent in real-world networks. While Turiyam Neutrosophic and Plithogenic Graphs are established within uncertain graph theory, concepts such as Line Graphs, Star Graphs, and Regular Graphs have not been extensively explored in these frameworks. This paper seeks to fill this gap by introducing and developing the concepts of Line Graph, Star Graph, and Regular Graph specifically within the contexts of Turiyam Neutrosophic and Plithogenic Graphs.

Keywords: Neutrosophic graph, Fuzzy graph, Plithogenic graph, Turiyam Neutrosophic Graph, Graph Class

1 | Introduction

1.1 | Graph Theory and Graph classes

Graph theory is a mathematical discipline studying structures called graphs, consisting of vertices (nodes) and edges (links) representing various relationships [1]. Many graph classes [2] have been defined in graph theory, including Unit Disk Graphs [3], Planar Graphs [4], and OuterPlanar Graphs [5]. These classes have well-established mathematical properties and applications. This paper focuses on three specific graph classes: Line Graphs, Star Graphs, and Regular Graphs.

A Line Graph is a graph where each vertex represents an edge of the original graph, with edges connecting vertices that share common endpoints [6]. A Star Graph is a tree structure with one central vertex connected to multiple peripheral vertices, forming a star-like shape [7]. A Regular Graph is characterized by uniform vertex degrees, where each vertex has the same number of adjacent edges [8].



Corresponding Author: Takaaki.fujita060@gmail.com



<https://doi.org/10.61356/j.plc.2025.4580>



Licensee Plithogenic Logic and Computation. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0>).

1.2 | Uncertain Graph Theory

This paper explores different models of uncertain graphs, such as Fuzzy[9], Intuitionistic Fuzzy[10], Neutrosophic[11], Turiyam Neutrosophic[12], and Plithogenic Graphs[13]. These models expand classical graph theory by incorporating various levels of uncertainty, making it easier to analyze complex and ambiguous relationships. Uncertain graph models have numerous real-world applications, which has led to the creation of many related graph classes [14]. Fundamental concepts like Fuzzy Sets and Neutrosophic Sets serve as the basis for these models and are widely covered in the literature [15].

Well-studied uncertain graph class concepts in this field include Line Graph, Star Graph, and Regular Graph. Notable variations are Fuzzy Line Graph [16], Fuzzy Star Graph [17], Fuzzy Regular Graph, Intuitionistic Fuzzy Line Graph [18], Neutrosophic Line Graph [19, 20], Neutrosophic Star Graph [21], and Neutrosophic Regular Graph [22]. Given the extensive literature and diverse applications, the study of uncertain graphs is highly significant. For a more detailed understanding, readers are encouraged to consult the existing survey papers [23].

1.3 | Our Contribution in This Paper

This Section outlines the importance of studying uncertain graphs and their various classes, which hold significance in both pure and applied mathematics. While Turiyam Neutrosophic and Plithogenic Graphs are known within the field of uncertain graph theory, concepts like Line Graphs, Star Graphs, and Regular Graphs remain relatively unexplored in these frameworks. This paper aims to address this gap by introducing and developing the concepts of Line Graph, Star Graph, and Regular Graph specifically within the contexts of Turiyam Neutrosophic and Plithogenic Graphs ¹. ²

2 | Preliminaries

This section provides an overview of the fundamental definitions and notations used throughout the paper. For a more detailed explanation of graph-theoretic notions and notation, see [25]. Note that in this paper, we use finite, undirected, and simple graphs.

2.1 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam Neutrosophic, and Plithogenic concepts. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" [26].

Definition 1 (Crisp Graph). (cf.[27]) A *crisp graph* is an ordered pair $G = (V, E)$, where:

- V is a finite, non-empty set of vertices.
- $E \subseteq V \times V$ is a set of edges, where each edge is an unordered pair of distinct vertices.

Formally, for any edge $(u, v) \in E$, the following holds:

$$(u, v) \in E \iff u \neq v \quad \text{and} \quad u, v \in V$$

This implies that there are no loops (i.e., no edges of the form (v, v)) and edges represent binary relationships between distinct vertices.

Example 2 (Crisp Graph). Consider the crisp graph $G = (V, E)$ defined as follows:

$$V = \{A, B, C, D\}, \quad E = \{(A, B), (B, C), (C, D), (A, D)\}.$$

Here, V is the set of vertices:

$$V = \{A, B, C, D\},$$

¹Turiyam Neutrosophic Graphs extend the traditional graph framework by incorporating four distinct membership values—truth, indeterminacy, falsity, and liberal state—for each vertex and edge. This approach enables a more nuanced representation of complex relationships [23].

²A Plithogenic Graph generalizes traditional graphs by incorporating multiple attributes and degrees of appurtenance, contradiction, and uncertainty for vertices and edges [24].

and E is the set of edges, each an unordered pair of distinct vertices. For instance, (A, B) indicates a connection between vertices A and B . By definition, there are no loops, so no pair of the form (A, A) exists.

This crisp graph can represent, for example, a simple network of four computers A, B, C, D connected by physical links $(A, B), (B, C), (C, D), (A, D)$. Each edge signifies a direct communication path between the corresponding computers.

Definition 3 (Unified Uncertain Graphs Framework). (cf.[14]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E . Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

(1) *Fuzzy Graph* [27, 28]:

- Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0, 1]$, representing the degree of participation of v in the fuzzy graph.
- Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0, 1]$, representing the strength of the connection between u and v .

(2) *Intuitionistic Fuzzy Graph (IFG)* [29]:

- Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0, 1]$ (degree of membership) and $\nu_A(v) \in [0, 1]$ (degree of non-membership), such that $\mu_A(v) + \nu_A(v) \leq 1$.
- Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0, 1]$ (degree of membership) and $\nu_B(u, v) \in [0, 1]$ (degree of non-membership), such that $\mu_B(u, v) + \nu_B(u, v) \leq 1$.

(3) *Neutrosophic Graph* [30]:

- Each vertex $v \in V$ is assigned a triple $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where:
 - $\sigma_T(v) \in [0, 1]$ is the truth-membership degree,
 - $\sigma_I(v) \in [0, 1]$ is the indeterminacy-membership degree,
 - $\sigma_F(v) \in [0, 1]$ is the falsity-membership degree,
 - $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
- Each edge $e = (u, v) \in E$ is assigned a triple $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$, representing the truth, indeterminacy, and falsity degrees for the connection between u and v .

(4) *Turiyam Neutrosophic Graph* [12]:

- Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where:
 - $t(v) \in [0, 1]$ is the truth value,
 - $iv(v) \in [0, 1]$ is the indeterminacy value,
 - $fv(v) \in [0, 1]$ is the falsity value,
 - $lv(v) \in [0, 1]$ is the liberal state value,
 - $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
- Each edge $e = (u, v) \in E$ is similarly assigned a quadruple representing the same parameters for the connection between u and v .

Example 4 (Fuzzy Graph in Project Management). Project Management coordinates resources, processes, and teams to achieve objectives within strict constraints of scope, time, budget, and quality, efficiently. Consider a project represented by a fuzzy graph $G_F = (V, E, \sigma, \mu)$. Let the set of vertices represent project tasks:

$$V = \{T_1, T_2, T_3, T_4\},$$

where

- T_1 : Requirement Analysis,
- T_2 : Design,
- T_3 : Implementation,
- T_4 : Testing.

Each task is assigned a membership degree indicating the confidence in its successful completion:

$$\sigma(T_1) = 0.9, \quad \sigma(T_2) = 0.8, \quad \sigma(T_3) = 0.7, \quad \sigma(T_4) = 0.85.$$

Let the set of edges represent dependencies between tasks:

$$E = \{(T_1, T_2), (T_2, T_3), (T_3, T_4)\}.$$

Each dependency is assigned a membership degree indicating the strength of the dependency:

$$\mu(T_1, T_2) = 0.95, \quad \mu(T_2, T_3) = 0.9, \quad \mu(T_3, T_4) = 0.85.$$

This fuzzy graph models the uncertainties in task execution and dependency reliability, providing a framework for effective project management.

Example 5 (Neutrosophic Graph in Social Network Analysis). Social Network Analysis investigates relationships among individuals, groups, or entities to uncover patterns, interactions, influence, and community structures within networks. Consider a social network modeled as a neutrosophic graph $G_N = (V, E)$. Let the set of vertices represent individuals:

$$V = \{P_1, P_2, P_3\}.$$

Each individual P_i is assigned a neutrosophic membership triple:

$$\sigma(P_i) = (\sigma_T(P_i), \sigma_I(P_i), \sigma_F(P_i)),$$

where:

$$\sigma(P_1) = (0.8, 0.1, 0.1), \quad \sigma(P_2) = (0.6, 0.2, 0.2), \quad \sigma(P_3) = (0.7, 0.15, 0.15).$$

Let the set of edges represent interpersonal connections, with each edge $e = (P_i, P_j)$ assigned a neutrosophic triple:

$$\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e)).$$

For instance, define:

$$\mu((P_1, P_2)) = (0.75, 0.15, 0.10), \quad \mu((P_2, P_3)) = (0.65, 0.20, 0.15), \quad \mu((P_3, P_1)) = (0.80, 0.10, 0.10).$$

This neutrosophic graph captures the nuances of trust, uncertainty, and distrust in social relationships, offering valuable insights into the dynamics of the network.

Definition 6 (Plithogenic Graph). (cf.[24]) Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A *Plithogenic Graph* PG is defined as:

$$PG = (PM, PN)$$

where:

(1) *Plithogenic Vertex Set* $PM = (M, l, Ml, adf, aCf)$:

- $M \subseteq V$ is the set of vertices.
- l is an attribute associated with the vertices.
- Ml is the range of possible attribute values.
- $adf : M \times Ml \rightarrow [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)* for vertices.
- $aCf : Ml \times Ml \rightarrow [0, 1]^t$ is the *Degree of Contradiction Function (DCF)* for vertices.

(2) *Plithogenic Edge Set* $PN = (N, m, Nm, bdf, bCf)$:

- $N \subseteq E$ is the set of edges.

- m is an attribute associated with the edges.
- Nm is the range of possible attribute values.
- $bdf : N \times Nm \rightarrow [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)* for edges.
- $bCf : Nm \times Nm \rightarrow [0, 1]^t$ is the *Degree of Contradiction Function (DCF)* for edges.

The Plithogenic Graph PG must satisfy the following conditions:

- (1) *Edge Appurtenance Constraint*: For all $(x, a), (y, b) \in M \times Ml$:

$$bdf((xy), (a, b)) \leq \min\{adf(x, a), adf(y, b)\}$$

where $xy \in N$ is an edge between vertices x and y , and $(a, b) \in Nm \times Nm$ are the corresponding attribute values.

- (2) *Contradiction Function Constraint*: For all $(a, b), (c, d) \in Nm \times Nm$:

$$bCf((a, b), (c, d)) \leq \min\{aCf(a, c), aCf(b, d)\}$$

- (3) *Reflexivity and Symmetry of Contradiction Functions*:

$$\begin{aligned} aCf(a, a) &= 0, & \forall a \in Ml \\ aCf(a, b) &= aCf(b, a), & \forall a, b \in Ml \\ bCf(a, a) &= 0, & \forall a \in Nm \\ bCf(a, b) &= bCf(b, a), & \forall a, b \in Nm \end{aligned}$$

The General Plithogenic Graph is a relax definition of the Plithogenic Graph (cf.[23, 31]).

Definition 7 (General Plithogenic Graph). [23] Let $G = (V, E)$ be a classical graph, where V is a finite set of vertices, and $E \subseteq V \times V$ is a set of edges.

A General Plithogenic Graph $G^{GP} = (PM, PN)$ consists of:

- (1) *General Plithogenic Vertex Set PM*:

$$PM = (M, l, Ml, adf, aCf)$$

where:

- $M \subseteq V$: Set of vertices.
- l : Attribute associated with the vertices.
- Ml : Range of possible attribute values.
- $adf : M \times Ml \rightarrow [0, 1]^s$: Degree of Appurtenance Function (DAF) for vertices.
- $aCf : Ml \times Ml \rightarrow [0, 1]^t$: Degree of Contradiction Function (DCF) for vertices.

- (2) *General Plithogenic Edge Set PN*:

$$PN = (N, m, Nm, bdf, bCf)$$

where:

- $N \subseteq E$: Set of edges.
- m : Attribute associated with the edges.
- Nm : Range of possible attribute values.
- $bdf : N \times Nm \rightarrow [0, 1]^s$: Degree of Appurtenance Function (DAF) for edges.
- $bCf : Nm \times Nm \rightarrow [0, 1]^t$: Degree of Contradiction Function (DCF) for edges.

The General Plithogenic Graph G^{GP} only needs to satisfy the following *Reflexivity and Symmetry* properties of the Contradiction Functions:

- Reflexivity and Symmetry of Contradiction Functions:

$$aCf(a, a) = 0, \quad \forall a \in Ml$$

$$aCf(a, b) = aCf(b, a), \quad \forall a, b \in Ml$$

$$bCf(a, a) = 0, \quad \forall a \in Nm$$

$$bCf(a, b) = bCf(b, a), \quad \forall a, b \in Nm$$

Example 8 (Real-life Example: Urban Transportation Network). An Urban Transportation Network comprises interconnected roads, rails, and public transit systems facilitating efficient movement of people, goods, and services [32]. Consider an urban transportation network modeled by a crisp graph

$$G = (V, E),$$

where the vertices represent intersections and the edges represent roads. Let

$$V = \{v_1, v_2, v_3, v_4\} \quad \text{and} \quad E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}.$$

Plithogenic Graph:

We construct a *Plithogenic Graph* $PG = (PM, PN)$ as follows.

Plithogenic Vertex Set $PM = (M, l, Ml, adf, aCf)$:

- Let $M = V$. Associate with each vertex an attribute l representing *Traffic Density* with the range of possible values

$$Ml = \{\text{Low, Medium, High}\}.$$

- Define the Degree of Appurtenance Function $adf : M \times Ml \rightarrow [0, 1]$ by, for example,

$$adf(v_1, \text{Low}) = 0.8,$$

$$adf(v_2, \text{Medium}) = 0.6,$$

$$adf(v_3, \text{High}) = 0.7,$$

$$adf(v_4, \text{Medium}) = 0.5.$$

- Define the Degree of Contradiction Function $aCf : Ml \times Ml \rightarrow [0, 1]$ by

$$aCf(\text{Low}, \text{Medium}) = 0.3,$$

$$aCf(\text{Low}, \text{High}) = 0.8,$$

$$aCf(\text{Medium}, \text{High}) = 0.5,$$

with $aCf(x, x) = 0$ for all $x \in Ml$ and $aCf(x, y) = aCf(y, x)$.

Plithogenic Edge Set $PN = (N, m, Nm, bdf, bCf)$:

- Let $N = E$. Associate with each edge an attribute m representing *Road Quality* with possible values

$$Nm = \{\text{Poor, Fair, Good}\}.$$

- Define the Degree of Appurtenance Function $bdf : N \times Nm \rightarrow [0, 1]$ by, for instance,

$$bdf((v_1, v_2), \text{Good}) = 0.9,$$

$$bdf((v_2, v_3), \text{Fair}) = 0.7,$$

$$bdf((v_3, v_4), \text{Good}) = 0.8,$$

$$bdf((v_4, v_1), \text{Fair}) = 0.6.$$

- Define the Degree of Contradiction Function $bCf : Nm \times Nm \rightarrow [0, 1]$ by

$$bCf(\text{Poor}, \text{Fair}) = 0.4,$$

$$bCf(\text{Poor}, \text{Good}) = 0.9,$$

$$bCf(\text{Fair}, \text{Good}) = 0.5,$$

with $bCf(x, x) = 0$ for all $x \in Nm$ and $bCf(x, y) = bCf(y, x)$.

This Plithogenic Graph $PG = (PM, PN)$ effectively models the uncertainties inherent in traffic density at intersections and road quality on the network.

A *General Plithogenic Graph* $G^{GP} = (PM, PN)$ is constructed similarly but with relaxed constraints. Using the same sets M, N, Ml , and Nm as above, we require only the reflexivity and symmetry of the contradiction functions aCf and bCf without enforcing the strict edge appurtenance constraint. This relaxation allows the model to accommodate additional uncertainties, such as temporary road closures or dynamic traffic conditions, thereby providing a more flexible framework for advanced traffic management systems.

2.2 | Uncertain Line Graph

The definitions of Fuzzy Line Graph and Neutrosophic Line Graph are provided below. These are extended graph concepts based on the Classic Line Graph.

Definition 9 (Fuzzy line graph). [16] A *fuzzy line graph* FL_n is a path with n vertices, of which exactly two vertices are leaves. In FL_n , the vertices are denoted as $u_1, u_2, \dots, u_n$, and the fuzzy degree of the edges is defined as follows:

$$\mu_i = \mu(u_i, u_{i+1}), \quad \text{for } 1 \leq i \leq n-1,$$

where $\mu_i > 0$ represents the fuzzy connection between u_i and u_{i+1} .

Example 10 (Fuzzy line graph). Consider a fuzzy line graph FL_4 with four vertices u_1, u_2, u_3, u_4 . The edges are $(u_1, u_2), (u_2, u_3), (u_3, u_4)$, and their fuzzy degrees are given as follows:

$$\mu(u_1, u_2) = 0.8, \quad \mu(u_2, u_3) = 0.6, \quad \mu(u_3, u_4) = 0.9.$$

The graph structure can be represented as:

$$u_1 \xrightarrow{0.8} u_2 \xrightarrow{0.6} u_3 \xrightarrow{0.9} u_4.$$

Here, $\mu(u_i, u_{i+1})$ indicates the strength of the connection between consecutive vertices. The sum of all fuzzy degrees gives a total "fuzzy weight" for the graph.

Definition 11 (Neutrosophic line graph). [19, 20] Let $G = (X_1, Y_1)$ be a *single-valued neutrosophic graph* (SVNG), where:

- $X_1 = \{\langle x, T_{X_1}(x), I_{X_1}(x), F_{X_1}(x) \rangle : x \in V\}$ is the single-valued neutrosophic vertex set.
- $Y_1 = \{\langle xy, T_{Y_1}(xy), I_{Y_1}(xy), F_{Y_1}(xy) \rangle : xy \in V \times V\}$ is the single-valued neutrosophic edge set, where $T_{Y_1}(xy)$, $I_{Y_1}(xy)$, and $F_{Y_1}(xy)$ are the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively.

The *neutrosophic line graph* $L(G)$ of G is defined as follows:

- (1) The vertex set of $L(G)$, denoted as X_2 , is given by:

$$X_2 = \{\langle S_x, T_{X_2}(S_x), I_{X_2}(S_x), F_{X_2}(S_x) \rangle : S_x \in Z\},$$

where $Z = \{\{x\} \cup \{u_x, v_x\} : x \in E, u_x, v_x \in V, x = u_x v_x\}$, and for each vertex $S_x \in X_2$:

$$T_{X_2}(S_x) = T_{Y_1}(x), \quad I_{X_2}(S_x) = I_{Y_1}(x), \quad F_{X_2}(S_x) = F_{Y_1}(x).$$

- (2) The edge set of $L(G)$, denoted as Y_2 , is given by:

$$Y_2 = \{\langle S_x S_y, T_{Y_2}(S_x S_y), I_{Y_2}(S_x S_y), F_{Y_2}(S_x S_y) \rangle : S_x S_y \in W\},$$

where $W = \{S_x S_y : S_x \cap S_y \neq \emptyset, x, y \in E, x \neq y\}$, and for each edge $S_x S_y \in Y_2$:

$$T_{Y_2}(S_x S_y) = \min\{T_{Y_1}(x), T_{Y_1}(y)\},$$

$$I_{Y_2}(S_x S_y) = \max\{I_{Y_1}(x), I_{Y_1}(y)\},$$

$$F_{Y_2}(S_x S_y) = \max\{F_{Y_1}(x), F_{Y_1}(y)\}.$$

Example 12 (Neutrosophic line graph). Consider a single-valued neutrosophic graph $G = (X_1, Y_1)$, where the vertices and edges have the following neutrosophic attributes:

$$X_1 = \{\langle v_1, 0.9, 0.1, 0.0 \rangle, \langle v_2, 0.8, 0.2, 0.0 \rangle, \langle v_3, 0.7, 0.3, 0.0 \rangle\}.$$

$$Y_1 = \{\langle e_1, 0.85, 0.1, 0.05 \rangle, \langle e_2, 0.75, 0.2, 0.05 \rangle\},$$

where $e_1 = (v_1, v_2)$ and $e_2 = (v_2, v_3)$. The neutrosophic line graph $L(G)$ has the following vertices and edges:

$$X_2 = \{\langle S_{e_1}, 0.85, 0.1, 0.05 \rangle, \langle S_{e_2}, 0.75, 0.2, 0.05 \rangle\}.$$

$$Y_2 = \{\langle S_{e_1} S_{e_2}, 0.75, 0.2, 0.05 \rangle\}.$$

Here, the truth, indeterminacy, and falsity memberships of the edges in $L(G)$ are derived based on the given formulas for T_{Y_2} , I_{Y_2} , and F_{Y_2} .

2.3 | Uncertain Star Graph

The definitions of Star Graph, Fuzzy Star Graph [17], and Neutrosophic Star Graph [21] are presented below.

Definition 13 (Star graph). [7, 33] A *star graph* is a type of tree graph that has one central node (also called the *hub*) and n peripheral nodes, where all the peripheral nodes are connected only to the central node.

- Formally, a star graph S_n can be defined as a graph with a vertex set

$$V = \{v_0, v_1, \dots, v_n\}$$

and an edge set

$$E = \{(v_0, v_i) : 1 \leq i \leq n\}.$$

- The central node v_0 is connected to each of the peripheral nodes v_i (for $1 \leq i \leq n$), and there are no edges between the peripheral nodes themselves.

Example 14 (Star graph). Consider a star graph S_4 with one central node v_0 and four peripheral nodes v_1, v_2, v_3, v_4 . The vertex set and edge set are:

$$V = \{v_0, v_1, v_2, v_3, v_4\}, \quad E = \{(v_0, v_1), (v_0, v_2), (v_0, v_3), (v_0, v_4)\}.$$

The graph has a tree-like structure with v_0 at the center connected to v_1, v_2, v_3 , and v_4 .

Definition 15 (Fuzzy star graph). [17] A *fuzzy star graph* FS_n consists of two sets of vertices, $V = \{v\}$ and $U = \{u_1, u_2, \dots, u_n\}$, satisfying the following conditions:

- There exists a fuzzy edge between the central vertex v and each surrounding vertex u_i , with fuzzy degree $\mu_i = \mu(v, u_i) > 0$ for $1 \leq i \leq n$.
- There is no fuzzy edge between any two surrounding vertices, i.e., $\mu(u_j, u_k) = 0$, where $1 \leq j, k \leq n$ and $j \neq k$.

Example 16 (Fuzzy star graph). Consider a fuzzy star graph FS_3 with one central vertex v and three surrounding vertices u_1, u_2, u_3 . The fuzzy degrees of the edges are:

$$\mu(v, u_1) = 0.7, \quad \mu(v, u_2) = 0.8, \quad \mu(v, u_3) = 0.6.$$

The edge set is:

$$E = \{(v, u_1), (v, u_2), (v, u_3)\}.$$

No edges exist between the vertices u_1, u_2, u_3 , i.e., $\mu(u_1, u_2) = \mu(u_1, u_3) = \mu(u_2, u_3) = 0$.

Definition 17 (Neutrosophic star graph). [21] Let $S_n = (V, E)$ be a *neutrosophic star graph* with:

- $V = \{v_1, v_2, \dots, v_n\}$, where v_1 is the *central vertex*.
- $E = \{e_j = v_1 v_j : 2 \leq j \leq n\}$, representing the edges connecting the central vertex v_1 to the other peripheral vertices v_j for $j = 2, \dots, n$.

The neutrosophic star graph S_n is characterized by three membership functions:

- (1) *Truth-membership function* $\sigma : V \rightarrow [0, 1]$ and $\alpha : E \rightarrow [0, 1]$.
- (2) *Indeterminacy-membership function* $\rho : V \rightarrow [0, 1]$ and $\beta : E \rightarrow [0, 1]$.
- (3) *Falsity-membership function* $\mu : V \rightarrow [0, 1]$ and $\gamma : E \rightarrow [0, 1]$.

The neutrosophic values for vertices and edges are defined as:

- For vertices v_j , $1 \leq j \leq n$:

$$\sigma(v_j) = \frac{4n - (j + (n - 1))}{2(2n + 1)}, \quad \rho(v_j) = \frac{2n - (j - 1)}{2(2n - 1)}, \quad \mu(v_j) = \frac{2n - (j - 1)}{3(2n - 1)}.$$

- For edges $e_j = v_1 v_j$, $2 \leq j \leq n$:

$$\alpha(e_j) = \frac{n - j}{2(2n + 1)}, \quad \beta(e_j) = \frac{n - j}{2(2n - 1)}, \quad \gamma(e_j) = \frac{n - j}{3(2n - 1)}.$$

Example 18 (Neutrosophic star graph). Consider a neutrosophic star graph S_3 with three vertices v_1, v_2, v_3 , where v_1 is the central vertex. The neutrosophic values for vertices and edges are as follows:

$$\sigma(v_1) = 0.9, \quad \rho(v_1) = 0.1, \quad \mu(v_1) = 0.0,$$

$$\sigma(v_2) = 0.8, \quad \rho(v_2) = 0.15, \quad \mu(v_2) = 0.05,$$

$$\sigma(v_3) = 0.7, \quad \rho(v_3) = 0.2, \quad \mu(v_3) = 0.1.$$

For edges:

$$\alpha(e_1) = 0.85, \quad \beta(e_1) = 0.1, \quad \gamma(e_1) = 0.05, \quad \text{for } e_1 = (v_1, v_2),$$

$$\alpha(e_2) = 0.75, \quad \beta(e_2) = 0.2, \quad \gamma(e_2) = 0.05, \quad \text{for } e_2 = (v_1, v_3).$$

The graph satisfies the neutrosophic membership conditions for vertices and edges.

2.4 | Uncertain Regular Graph

The definitions of Regular Graph, Regular Fuzzy Graph [34], and Regular Neutrosophic Graph[22] are presented below.

Definition 19 (Regular graph). A *regular graph* is a graph in which every vertex has the same degree.

- Formally, a graph $G = (V, E)$ is called *k-regular* if for every vertex $v \in V$, the degree $d(v)$ (i.e., the number of edges incident to v) is equal to k .
- A *k-regular graph* has a vertex set V and an edge set E , satisfying

$$d(v) = k, \quad \text{for all } v \in V.$$

Example 20 (Regular graph). Consider a 3-regular graph $G = (V, E)$ with the following structure:

$$V = \{v_1, v_2, v_3, v_4\}, \quad E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}.$$

Each vertex in this graph has exactly three edges incident to it. For example:

$$d(v_1) = 3, \quad d(v_2) = 3, \quad d(v_3) = 3, \quad d(v_4) = 3.$$

This graph satisfies the property of being 3-regular.

Definition 21 (Regular fuzzy graph). [34, 35] A *regular fuzzy graph* is a fuzzy graph $G = (\sigma, \mu)$, where:

- $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of the vertex set V .
- $\mu : V \times V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ , defining the fuzzy degree of edges.

The *degree* of a vertex u in G , denoted as $d_G(u)$, is defined by:

$$d_G(u) = \sum_{u \neq v} \mu(u, v) = \sum_{uv \in E} \mu(u, v).$$

A fuzzy graph G is called *regular of degree k* (or a *k -regular fuzzy graph*) if every vertex has the same degree k . That is, for all $u \in V$,

$$d_G(u) = k.$$

Equivalently, the *minimum degree* $\delta(G)$ and the *maximum degree* $\Delta(G)$ of the fuzzy graph are both equal to k , i.e.,

$$\delta(G) = \Delta(G) = k.$$

Example 22 (Regular fuzzy graph). Consider a fuzzy graph $G = (\sigma, \mu)$ with the following properties:

$$\begin{aligned} V &= \{v_1, v_2, v_3\}, \quad \sigma(v_1) = 1, \quad \sigma(v_2) = 1, \quad \sigma(v_3) = 1, \\ \mu(v_1, v_2) &= 0.5, \quad \mu(v_1, v_3) = 0.5, \quad \mu(v_2, v_3) = 0.5. \end{aligned}$$

The degree of each vertex is:

$$d_G(v_1) = 0.5 + 0.5 = 1.0, \quad d_G(v_2) = 0.5 + 0.5 = 1.0, \quad d_G(v_3) = 0.5 + 0.5 = 1.0.$$

Thus, this fuzzy graph is a 1-regular fuzzy graph.

Definition 23 (Regular Neutrosophic Graph). [22] A *Regular Neutrosophic Graph* is a type of single-valued neutrosophic graph (SVNG), defined by the following characteristics:

(1) Let $G = (N, M)$ be an SVNG, where:

- N is the set of vertices with neutrosophic attributes represented by three membership functions:
 - Truth-membership $T_N : V \rightarrow [0, 1]$,
 - Indeterminacy-membership $I_N : V \rightarrow [0, 1]$,
 - Falsity-membership $F_N : V \rightarrow [0, 1]$.
- M is the set of edges with neutrosophic attributes represented by three membership functions:
 - Truth-membership $T_M : E \rightarrow [0, 1]$,
 - Indeterminacy-membership $I_M : E \rightarrow [0, 1]$,
 - Falsity-membership $F_M : E \rightarrow [0, 1]$.

(2) A *regular neutrosophic graph* is defined such that for every node $a \in V$, the sum of the truth, indeterminacy, and falsity membership values of all edges incident to a is constant across the graph:

$$\sum_{a \neq b} T_M(a, b) = \text{Constant}, \quad \sum_{a \neq b} I_M(a, b) = \text{Constant}, \quad \sum_{a \neq b} F_M(a, b) = \text{Constant}$$

where $(a, b) \in E$ represents an edge in the graph.

(3) The *degree of a node* $a \in V$ in a regular neutrosophic graph is given by the tuple $d(a) = (d_T(a), d_I(a), d_F(a))$, where:

$$d_T(a) = \sum_{a \neq b} T_M(a, b), \quad d_I(a) = \sum_{a \neq b} I_M(a, b), \quad d_F(a) = \sum_{a \neq b} F_M(a, b)$$

The degrees $d_T(a)$, $d_I(a)$, and $d_F(a)$ represent the sum of the truth, indeterminacy, and falsity memberships of all edges incident to a , respectively.

Example 24 (Regular neutrosophic graph). Consider a neutrosophic graph $G = (V, E)$ with:

$$V = \{v_1, v_2, v_3\}, \quad E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}.$$

The neutrosophic membership values for edges are:

$$\begin{aligned} T_M(v_1, v_2) &= 0.6, & T_M(v_1, v_3) &= 0.4, & T_M(v_2, v_3) &= 0.5, \\ I_M(v_1, v_2) &= 0.3, & I_M(v_1, v_3) &= 0.3, & I_M(v_2, v_3) &= 0.4, \\ F_M(v_1, v_2) &= 0.1, & F_M(v_1, v_3) &= 0.3, & F_M(v_2, v_3) &= 0.1. \end{aligned}$$

The degree of v_1 is:

$$d_T(v_1) = 0.6 + 0.4 = 1.0, \quad d_I(v_1) = 0.3 + 0.3 = 0.6, \quad d_F(v_1) = 0.1 + 0.3 = 0.4.$$

Similarly, the degrees of v_2 and v_3 are computed, and all are equal. Thus, G is a regular neutrosophic graph.

3 | Result in this paper: Some Graph Class

The results of this paper are presented below.

3.1 | Turiyam Neutrosophic Line Graph and General Plithogenic Line Graph

The definitions of Turiyam Neutrosophic Line Graph and General Plithogenic Line Graph, as well as their relationships with other graph classes, are described below.

Definition 25 (Turiyam Neutrosophic Line Graph). Given a Turiyam Neutrosophic Graph $G = (V, E)$, the *Turiyam Neutrosophic Line Graph* $L(G)$ is defined as follows:

- The vertex set V' of $L(G)$ corresponds to the edge set E of G :

$$V' = \{v_e : e \in E\}$$

- Two vertices v_{e_1} and v_{e_2} in V' are adjacent in $L(G)$ if and only if the edges e_1 and e_2 in G are adjacent.
- The membership degrees of $v_e \in V'$ are defined as:

$$\sigma(v_e) = \mu(e)$$

Example 26 (Real-life Example of a Turiyam Neutrosophic Line Graph: Railway Route). Consider a railway transportation system where stations are vertices and tracks are edges. Uncertainty arises due to delays, maintenance, or unpredictable events. We model this as a *Turiyam Neutrosophic Line Graph*.

Let the line graph represent a simple railway route with four stations:

$$V = \{A, B, C, D\}, \quad E = \{(A, B), (B, C), (C, D)\}.$$

Each edge represents a railway segment and is assigned a quadruple of Turiyam neutrosophic values (t, iv, fv, lv) capturing truth (on-time probability), indeterminacy (uncertainty of schedule), falsity (chance of disruption), and liberal state (flexibility of alternative options):

$$\mu(A, B) = (0.85, 0.10, 0.03, 0.02), \quad \mu(B, C) = (0.80, 0.15, 0.02, 0.03), \quad \mu(C, D) = (0.75, 0.15, 0.05, 0.05).$$

Thus, the Turiyam Neutrosophic Line Graph models railway reliability where each edge represents not only the physical connection but also nuanced real-world uncertainties.

Definition 27 (General Plithogenic Line Graph). Given a General Plithogenic Graph $G^{GP} = (PM, PN)$ with vertex set V and edge set E , the *General Plithogenic Line Graph* $L(G^{GP}) = (PM', PN')$ is defined as follows:

(1) *Vertex Set* PM' :

- The vertices of $L(G^{GP})$ correspond to the edges of G^{GP} :

$$V' = \{v_e : e \in E\}$$

- Attribute for vertices:

$$l'(v_e) = m(e)$$

- Possible attribute values:

$$Ml' = Nm$$

- Degree of Appurtenance Function:

$$\text{adf}'(v_e, a) = \text{bdf}(e, a), \quad \forall a \in Ml'$$

- Degree of Contradiction Function:

$$\text{aCf}'(a, b) = \text{bCf}(a, b), \quad \forall a, b \in Ml'$$

(2) *Edge Set PN'*:

- Edges in $L(G^{GP})$:

$$E' = \{(v_{e_1}, v_{e_2}) : e_1, e_2 \in E, e_1 \cap e_2 \neq \emptyset\}$$

- Attribute for edges:

$$m'(v_{e_1}, v_{e_2}) = (l(u), l(v)), \quad \text{where } e_1 = (u, w), e_2 = (w, v)$$

- Possible attribute values:

$$Nm' = Ml \times Ml$$

- Degree of Appurtenance Function:

$$\text{bdf}'((v_{e_1}, v_{e_2}), b) = f(\text{adf}(u, l(u)), \text{adf}(v, l(v))), \quad \forall b \in Nm'$$

- Degree of Contradiction Function:

$$\text{bCf}'(b_1, b_2) = \text{aCf}(a_1, a_2), \quad \text{for } b_i = (a_i^1, a_i^2)$$

Example 28 (General Plithogenic Line Graph: Power Distribution Feeder). We model a radial power feeder with substations as vertices and cables as edges. Let the underlying crisp graph be

$$V = \{S_1, S_2, S_3, S_4\}, \quad E = \{e_1 = (S_1, S_2), e_2 = (S_2, S_3), e_3 = (S_3, S_4)\}.$$

Define a General Plithogenic Graph $G^{GP} = (PM, PN)$ as follows.

Vertex attribute (load class): $l : V \rightarrow Ml$, with

$$Ml = \{\text{Low, Med, High}\}, \quad l(S_1) = \text{Low}, l(S_2) = \text{Med}, l(S_3) = \text{High}, l(S_4) = \text{Med}.$$

Vertex DAF dimension $s = 2$, interpreted as (membership, stability):

$$\begin{aligned} \text{adf}(S_1, \text{Low}) &= (0.80, 0.70), & \text{adf}(S_2, \text{Med}) &= (0.70, 0.60), \\ \text{adf}(S_3, \text{High}) &= (0.60, 0.50), & \text{adf}(S_4, \text{Med}) &= (0.75, 0.65). \end{aligned}$$

Edge attribute (cable health): $m : E \rightarrow Nm$ with

$$Nm = \{\text{Old, Std, New}\}, \quad m(e_1) = \text{Std}, m(e_2) = \text{Old}, m(e_3) = \text{New}.$$

Edge DAF $\text{bdf} : N \times Nm \rightarrow [0, 1]^2$ (chosen concretely):

$$\text{bdf}(e_1, \text{Std}) = (0.70, 0.80), \quad \text{bdf}(e_2, \text{Old}) = (0.50, 0.60), \quad \text{bdf}(e_3, \text{New}) = (0.90, 0.90).$$

Contradiction functions (only reflexive/symmetric are required in the general model):

$$\begin{aligned} \text{aCf}(x, x) &= 0, \quad \text{aCf}(x, y) = \text{aCf}(y, x), \quad \text{aCf}(\text{Low, Med}) = 0.30, \\ \text{aCf}(\text{Low, High}) &= 0.70, \quad \text{aCf}(\text{Med, High}) = 0.40; \\ \text{bCf}(z, z) &= 0, \quad \text{bCf}(z, z') = \text{bCf}(z', z), \quad \text{bCf}(\text{Old, Std}) = 0.40, \\ \text{bCf}(\text{Old, New}) &= 0.80, \quad \text{bCf}(\text{Std, New}) = 0.50. \end{aligned}$$

Line graph $L(G^{GP}) = (PM', PN')$:

$$V' = \{v_{e_1}, v_{e_2}, v_{e_3}\}, \quad E' = \{(v_{e_1}, v_{e_2}), (v_{e_2}, v_{e_3})\}.$$

By definition,

$$l'(v_e) = m(e), \quad Ml' = Nm, \quad \text{adf}'(v_e, a) := \text{bdf}(e, a).$$

Hence

$$\begin{aligned} l'(v_{e_1}) &= \text{Std}, \quad \text{adf}'(v_{e_1}, \text{Std}) = (0.70, 0.80), \\ l'(v_{e_2}) &= \text{Old}, \quad \text{adf}'(v_{e_2}, \text{Old}) = (0.50, 0.60), \\ l'(v_{e_3}) &= \text{New}, \quad \text{adf}'(v_{e_3}, \text{New}) = (0.90, 0.90). \end{aligned}$$

For edges of $L(G^{GP})$, set $m'(v_{e_1}, v_{e_2}) = (l(S_1), l(S_3)) = (\text{Low}, \text{High})$ and $m'(v_{e_2}, v_{e_3}) = (l(S_2), l(S_4)) = (\text{Med}, \text{Med})$. With the concrete choice $f = \min$ (componentwise),

$$\begin{aligned} \text{bdf}'((v_{e_1}, v_{e_2}), (\text{Low}, \text{High})) &= \min(\text{adf}(S_1, \text{Low}), \text{adf}(S_3, \text{High})) = (0.60, 0.50), \\ \text{bdf}'((v_{e_2}, v_{e_3}), (\text{Med}, \text{Med})) &= \min(\text{adf}(S_2, \text{Med}), \text{adf}(S_4, \text{Med})) = (0.70, 0.60). \end{aligned}$$

Finally set $aCf' = bCf$ on $Ml' = Nm$ (reflexive/symmetric). This yields a fully specified *General Plithogenic Line Graph* reflecting feeder segments and their health.

Theorem 29. *Let $L(G^{GP}) = (PM', PN')$ be a General Plithogenic Line Graph with a degree of appurtenance function dimension s and a degree of contradiction function dimension t . Then:*

- (1) *By setting $s = 4$ and defining suitable mappings, $L(G^{GP})$ can be transformed into a Turiyam Neutrosophic Line Graph.*
- (2) *By setting $s = 3$ and defining suitable mappings, $L(G^{GP})$ can be transformed into a Neutrosophic Line Graph.*
- (3) *By setting $s = 1$ and defining suitable mappings, $L(G^{GP})$ can be transformed into a Fuzzy Line Graph.*

Proof: To prove this theorem, we divide the proof into three cases based on the value of s : $s = 4$, $s = 3$, and $s = 1$.

First, consider the case where $s = 4$. By definition, a Turiyam Neutrosophic Line Graph is characterized by a membership vector for each vertex and edge, containing four components: truth, indeterminacy, falsity, and liberal state. For each vertex v_e in $L(G^{GP})$, we define its membership vector as

$$\sigma(v_e) = (T(v_e), I(v_e), F(v_e), L(v_e)) = \text{adf}'(v_e, l'(v_e)).$$

Here, $T(v_e)$, $I(v_e)$, $F(v_e)$, and $L(v_e)$ correspond to the truth, indeterminacy, falsity, and liberal states of v_e , respectively. Similarly, for each edge $e' = (v_{e_1}, v_{e_2})$, its membership vector is defined as

$$\mu(e') = (T(e'), I(e'), F(e'), L(e')) = \text{bdf}'(e', m'(e')).$$

The validity of this construction is ensured by the condition

$$T(v_e) + I(v_e) + F(v_e) + L(v_e) \leq 1,$$

which holds for all vertices v_e . This confirms that $L(G^{GP})$ satisfies the properties of a Turiyam Neutrosophic Line Graph.

Next, consider the case where $s = 3$. By definition, a Neutrosophic Line Graph is characterized by a membership vector for each vertex and edge, containing three components: truth, indeterminacy, and falsity. For each vertex v_e in $L(G^{GP})$, its membership vector is defined as

$$\sigma(v_e) = (T(v_e), I(v_e), F(v_e)) = \text{adf}'(v_e, l'(v_e)).$$

Here, $T(v_e)$, $I(v_e)$, and $F(v_e)$ represent the truth, indeterminacy, and falsity states of v_e , respectively. Similarly, for each edge $e' = (v_{e_1}, v_{e_2})$, its membership vector is defined as

$$\mu(e') = (T(e'), I(e'), F(e')) = \text{bdf}'(e', m'(e')).$$

The validity of this construction is ensured by the condition

$$T(v_e) + I(v_e) + F(v_e) \leq 1,$$

which holds for all vertices v_e . This confirms that $L(G^{GP})$ satisfies the properties of a Neutrosophic Line Graph. Finally, consider the case where $s = 1$. By definition, a Fuzzy Line Graph is characterized by a single-valued membership degree for each vertex and edge. For each vertex v_e in $L(G^{GP})$, its membership degree is defined as

$$\sigma(v_e) = \text{adf}'(v_e, l'(v_e)).$$

Here, $\sigma(v_e)$ represents the single-valued membership degree of v_e . Similarly, for each edge $e' = (v_{e_1}, v_{e_2})$, its membership degree is defined as

$$\mu(e') = \text{bdf}'(e', m'(e')).$$

The validity of this construction is ensured by the condition

$$\sigma(v_e) \leq 1,$$

which holds for all vertices v_e . This confirms that $L(G^{GP})$ satisfies the properties of a Fuzzy Line Graph.

In conclusion, the proof demonstrates that by varying the dimension s of the degree of appurtenance function and defining suitable mappings, the General Plithogenic Line Graph $L(G^{GP})$ can be transformed into a Turiyam Neutrosophic Line Graph ($s = 4$), a Neutrosophic Line Graph ($s = 3$), or a Fuzzy Line Graph ($s = 1$). $\square$

3.2 | Turiyam Neutrosophic Star Graph and General Plithogenic Star Graph

The definitions of Turiyam Neutrosophic Star Graph and General Plithogenic Star Graph are provided below.

Definition 30 (Turiyam Neutrosophic Star Graph). A *Turiyam Neutrosophic Star Graph* $S_n = (V, E)$ is a Turiyam Neutrosophic Graph where:

- $V = \{v_0, v_1, v_2, \dots, v_n\}$, with v_0 being the central vertex.
- $E = \{e_i = (v_0, v_i) : 1 \leq i \leq n\}$.

Example 31 (Real-life Example of a Turiyam Neutrosophic Star Graph: Airport Hub System). Consider an airport hub network where one central airport (hub) connects to multiple regional airports. Uncertainties exist due to weather, delays, or operational restrictions. We model this as a *Turiyam Neutrosophic Star Graph*.

Let

$$V = \{v_0, v_1, v_2, v_3\},$$

where v_0 is the central international airport, and v_1, v_2, v_3 are regional airports. The edge set is

$$E = \{(v_0, v_1), (v_0, v_2), (v_0, v_3)\}.$$

Each edge is assigned a quadruple (t, iv, fv, lv) :

$$\mu(v_0, v_1) = (0.90, 0.05, 0.03, 0.02), \quad \mu(v_0, v_2) = (0.80, 0.10, 0.05, 0.05), \quad \mu(v_0, v_3) = (0.70, 0.15, 0.10, 0.05).$$

Here:

- t : probability of on-time connectivity,
- iv : uncertainty due to external factors,
- fv : probability of failure or cancellation,
- lv : liberal state reflecting flexibility for rerouting.

This Turiyam Neutrosophic Star Graph effectively represents an airport hub system with real-world uncertainties.

Definition 32 (General Plithogenic Star Graph). A *General Plithogenic Star Graph* $S_n^{GP} = (PM, PN)$ is defined as:

(1) *Vertex Set PM*:

- $V = \{v_0, v_1, v_2, \dots, v_n\}$, with v_0 as the central vertex.

- Attributes:

$$l(v_i) \in Ml, \quad \forall i = 0, 1, \dots, n$$

- Degree of Appurtenance Function:

$$\text{adf}(v_i, a) \in [0, 1]^s, \quad \forall a \in Ml$$

(2) *Edge Set PN*:

- $E = \{e_i = (v_0, v_i) : 1 \leq i \leq n\}$

- Attributes:

$$m(e_i) \in Nm$$

- Degree of Appurtenance Function:

$$\text{bdf}(e_i, b) = f(\text{adf}(v_0, l(v_0)), \text{adf}(v_i, l(v_i))), \quad \forall b \in Nm$$

Example 33 (General Plithogenic Star Graph: Warehouse-to-Stores Logistics). Consider a distribution hub w_0 serving three retail stores s_1, s_2, s_3 . Let $V = \{w_0, s_1, s_2, s_3\}$ and $E = \{e_1 = (w_0, s_1), e_2 = (w_0, s_2), e_3 = (w_0, s_3)\}$.

Vertex attribute (demand level): $Ml = \{\text{Low}, \text{Med}, \text{High}\}$ with

$$l(w_0) = \text{High}, \quad l(s_1) = \text{Med}, \quad l(s_2) = \text{Low}, \quad l(s_3) = \text{Med}.$$

Take $s = 2$ and specify

$$\begin{aligned} \text{adf}(w_0, \text{High}) &= (0.90, 0.80), & \text{adf}(s_1, \text{Med}) &= (0.70, 0.70), \\ \text{adf}(s_2, \text{Low}) &= (0.50, 0.90), & \text{adf}(s_3, \text{Med}) &= (0.65, 0.75). \end{aligned}$$

Edge attribute (road type): $Nm = \{\text{Highway}, \text{City}\}$ with

$$m(e_1) = \text{Highway}, \quad m(e_2) = \text{City}, \quad m(e_3) = \text{Highway}.$$

Define $\text{bdf}(e, b) = \min(\text{adf}(\text{end}_1, l(\text{end}_1)), \text{adf}(\text{end}_2, l(\text{end}_2)))$ (componentwise). Then

$$\begin{aligned} \text{bdf}(e_1, \text{Highway}) &= \min((0.90, 0.80), (0.70, 0.70)) = (0.70, 0.70), \\ \text{bdf}(e_2, \text{City}) &= \min((0.90, 0.80), (0.50, 0.90)) = (0.50, 0.80), \\ \text{bdf}(e_3, \text{Highway}) &= \min((0.90, 0.80), (0.65, 0.75)) = (0.65, 0.75). \end{aligned}$$

Choose contradiction functions with reflexivity/symmetry (general model):

$$\begin{aligned} \text{acf}(\text{Low}, \text{Low}) &= \text{acf}(\text{Med}, \text{Med}) = \text{acf}(\text{High}, \text{High}) = 0, \\ \text{acf}(\text{Low}, \text{Med}) &= 0.30, \quad \text{acf}(\text{Low}, \text{High}) = 0.70, \quad \text{acf}(\text{Med}, \text{High}) = 0.40, \\ \text{bcf}(\text{Highway}, \text{Highway}) &= \text{bcf}(\text{City}, \text{City}) = 0, \quad \text{bcf}(\text{Highway}, \text{City}) = 0.60. \end{aligned}$$

This yields a *General Plithogenic Star Graph* where edges encode the serviceability of routes given hub/store demand and road type.

Theorem 34. *Similar to Theorem 29, applied to Star Graphs.*

Proof: Let $S_n^{GP} = (PM, PN)$ be a General Plithogenic Star Graph with a degree of appurtenance function dimension s and a degree of contradiction function dimension t . We prove the theorem in three cases.

Case 1: Transformation into a Turiyam Neutrosophic Star Graph

Assume $s = 4$. Define the membership vectors for vertices and edges as follows. For each vertex v_i ,

$$\sigma(v_i) = (T(v_i), I(v_i), F(v_i), L(v_i)) = \text{adf}(v_i, l(v_i)),$$

and for each edge $e_i = (v_0, v_i)$,

$$\mu(e_i) = (T(e_i), I(e_i), F(e_i), L(e_i)) = \text{bdf}(e_i, m(e_i)).$$

Verify that

$$T(v_i) + I(v_i) + F(v_i) + L(v_i) \leq 1, \quad \forall v_i \in V,$$

and

$$T(e_i) + I(e_i) + F(e_i) + L(e_i) \leq 1, \quad \forall e_i \in E.$$

Thus, S_n^{GP} satisfies the properties of a Turiyam Neutrosophic Star Graph.

Case 2: Transformation into a Neutrosophic Star Graph

Assume $s = 3$. Define the membership vectors for vertices and edges as follows. For each vertex v_i ,

$$\sigma(v_i) = (T(v_i), I(v_i), F(v_i)) = \text{adf}(v_i, l(v_i)),$$

and for each edge $e_i = (v_0, v_i)$,

$$\mu(e_i) = (T(e_i), I(e_i), F(e_i)) = \text{bdf}(e_i, m(e_i)).$$

Verify that

$$T(v_i) + I(v_i) + F(v_i) \leq 1, \quad \forall v_i \in V,$$

and

$$T(e_i) + I(e_i) + F(e_i) \leq 1, \quad \forall e_i \in E.$$

Thus, S_n^{GP} satisfies the properties of a Neutrosophic Star Graph.

Case 3: Transformation into a Fuzzy Star Graph

Assume $s = 1$. Define the membership degrees for vertices and edges as follows. For each vertex v_i ,

$$\sigma(v_i) = \text{adf}(v_i, l(v_i)),$$

and for each edge $e_i = (v_0, v_i)$,

$$\mu(e_i) = \text{bdf}(e_i, m(e_i)).$$

Verify that

$$\sigma(v_i) \leq 1, \quad \forall v_i \in V,$$

and

$$\mu(e_i) \leq 1, \quad \forall e_i \in E.$$

Thus, S_n^{GP} satisfies the properties of a Fuzzy Star Graph. $\square$

3.3 | Regular Turiyam Neutrosophic Graph and General Regular Plithogenic Graph

The definitions of Regular Turiyam Neutrosophic Graph and General Regular Plithogenic Graph, as well as their relationships with other graph classes, are briefly discussed.

Definition 35 (Regular Turiyam Neutrosophic Graph). A *Regular Turiyam Neutrosophic Graph* is a Turiyam Neutrosophic Graph where, for every vertex $v \in V$, the sum of the membership degrees of all edges incident to v is constant across the graph. That is, there exist constants k_T, k_I, k_F, k_L such that:

$$\sum_{u \in V, u \neq v} T(v, u) = k_T, \quad \sum_{u \in V, u \neq v} I(v, u) = k_I, \quad \sum_{u \in V, u \neq v} F(v, u) = k_F, \quad \sum_{u \in V, u \neq v} L(v, u) = k_L$$

Example 36 (Regular Turiyam Neutrosophic Graph: Symmetric Communication Triangle). Let $G = (V, E)$ be a triangle of communication links:

$$V = \{A, B, C\}, \quad E = \{AB, BC, CA\}.$$

Assign to each edge the same Turiyam neutrosophic quadruple (T, I, F, L) capturing (on-time success, indeterminacy, failure, liberal/flex reserve):

$$\mu(AB) = \mu(BC) = \mu(CA) = (0.60, 0.20, 0.10, 0.10).$$

For each vertex v , define the (Turiyam) degree-sums

$$d_T(v) = \sum_{u \neq v} T(v, u), \quad d_I(v) = \sum_{u \neq v} I(v, u), \quad d_F(v) = \sum_{u \neq v} F(v, u), \quad d_L(v) = \sum_{u \neq v} L(v, u).$$

Since each v is incident to exactly two edges with identical values, we have

$$\begin{aligned} d_T(A) &= d_T(B) = d_T(C) = 0.60 + 0.60 = 1.20, \\ d_I(A) &= d_I(B) = d_I(C) = 0.20 + 0.20 = 0.40, \\ d_F(A) &= d_F(B) = d_F(C) = 0.10 + 0.10 = 0.20, \\ d_L(A) &= d_L(B) = d_L(C) = 0.10 + 0.10 = 0.20. \end{aligned}$$

Thus the four component-sums are constant across all vertices; hence G is a *Regular Turiyam Neutrosophic Graph*.

Definition 37 (General Regular Plithogenic Graph). A *General Regular Plithogenic Graph* $G^{GP} = (PM, PN)$ is one where, for every vertex $v \in V$, the sum of the DAFs of all edges incident to v is constant across the graph. For each component $k = 1, \dots, s$, there exist constants K_k such that:

$$\sum_{u \in V, u \neq v} \text{bdf}_k((v, u), m(v, u)) = K_k, \quad \forall v \in V$$

Example 38 (General Regular Plithogenic Graph: Balanced Enterprise LAN Segment). Consider a square LAN segment

$$V = \{v_1, v_2, v_3, v_4\}, \quad E = \{e_{12}, e_{23}, e_{34}, e_{41}\},$$

with edges $e_{ij} = (v_i, v_j)$. Let the vertex attribute (role) be $MI = \{\text{Core}, \text{Dist}, \text{Access}\}$ with some assignment $l(v_i)$ (arbitrary here), and choose any vertex DAF adf (not used for regularity).

For edges, take $s = 3$ components in bdf (e.g., throughput, reliability, compliance). Let the edge attribute be $Nm = \{\text{Fiber}, \text{Copper}, \text{Wireless}\}$ with

$$m(e_{12}) = \text{Fiber}, \quad m(e_{23}) = \text{Copper}, \quad m(e_{34}) = \text{Wireless}, \quad m(e_{41}) = \text{Fiber}.$$

Specify the *same* edge DAF vector for all edges (permitted in the general model):

$$\text{bdf}(e, m(e)) = (0.60, 0.40, 0.30) \quad \text{for each } e \in E.$$

Then each vertex has degree 2, so the per-component sums at every vertex are

$$K = (K_1, K_2, K_3) = 2 \cdot (0.60, 0.40, 0.30) = (1.20, 0.80, 0.60),$$

which are independent of the vertex. Therefore, this is a *General Regular Plithogenic Graph* with constants

$$\sum_{u \neq v} \text{bdf}_k((v, u), m(v, u)) = K_k \quad (k = 1, 2, 3), \quad \text{for all } v \in V.$$

Choose contradiction functions reflexive/symmetric (as required only):

$$aCf(x, x) = 0, \quad aCf(x, y) = aCf(y, x); \quad bCf(z, z) = 0, \quad bCf(z, z') = bCf(z', z).$$

This construction captures a balanced LAN ring where each switch sees the same aggregate (throughput, reliability, compliance) regardless of its position.

Theorem 39. *Similar to Theorem 29, applied to Regular Graphs.*

Proof: Let $G^{GP} = (PM, PN)$ be a General Regular Plithogenic Graph with a degree of appurtenance function dimension s and a degree of contradiction function dimension t . We prove the theorem in three cases.

Case 1: Transformation into a Regular Turiyam Neutrosophic Graph

Assume $s = 4$. Define the membership vectors for edges as follows. For each edge $e = (v, u)$,

$$\mu(e) = (T(e), I(e), F(e), L(e)) = \text{bdf}(e, m(e)).$$

To ensure that G^{GP} satisfies the properties of a Regular Turiyam Neutrosophic Graph, the following conditions must hold:

$$\sum_{u \in V, u \neq v} T(v, u) = k_T, \quad \sum_{u \in V, u \neq v} I(v, u) = k_I,$$

$$\sum_{u \in V, u \neq v} F(v, u) = k_F, \quad \sum_{u \in V, u \neq v} L(v, u) = k_L.$$

Additionally, verify that for each edge e :

$$T(e) + I(e) + F(e) + L(e) \leq 1.$$

Case 2: Transformation into a Regular Neutrosophic Graph

Assume $s = 3$. For each edge $e = (v, u)$, define the membership vector as:

$$\mu(e) = (T(e), I(e), F(e)) = \text{bdf}(e, m(e)).$$

Ensure the regularity conditions:

$$\sum_{u \in V, u \neq v} T(v, u) = k_T, \quad \sum_{u \in V, u \neq v} I(v, u) = k_I, \quad \sum_{u \in V, u \neq v} F(v, u) = k_F.$$

Verify that for each edge e :

$$T(e) + I(e) + F(e) \leq 1.$$

Case 3: Transformation into a Regular Fuzzy Graph

Assume $s = 1$. For each edge $e = (v, u)$, define the membership degree as:

$$\mu(e) = \text{bdf}(e, m(e)).$$

Ensure the regularity condition:

$$\sum_{u \in V, u \neq v} \mu(v, u) = k.$$

Verify that for each edge e :

$$\mu(e) \leq 1.$$

□

4 | Conclusion and Future Tasks

This paper introduced the concepts of Line Graph, Star Graph, and Regular Graph specifically within the contexts of Turiyam Neutrosophic and Plithogenic Graphs. Future research directions involve extending the aforementioned graphs to hypergraphs [36, 37] and superhypergraphs [38, 39, 40]. Additionally, we plan to study graph algorithms and their fuzzification.

Funding

This research received no external funding.

Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

- [1] Reinhard Diestel. Graduate texts in mathematics: Graph theory.
- [2] Andreas Brandstädt, Van Bang Le, and Jeremy P Spinrad. *Graph classes: a survey*. SIAM, 1999.
- [3] Aistis Atminas and Viktor Zamaraev. On forbidden induced subgraphs for unit disk graphs. *Discrete & Computational Geometry*, 60(1):57–97, 2018.
- [4] Carsten R Seemann, Vincent Moulton, Peter F Stadler, and Marc Hellmuth. Planar median graphs and cubesquare-graphs. *Discrete Applied Mathematics*, 331:38–58, 2023.
- [5] Therese Biedl. On triangulating k-outerplanar graphs. *Discrete Applied Mathematics*, 181:275–279, 2015.
- [6] Ladislav Nebeský. On the line graph of the square and the square of the line graph of a connected graph. *Časopis pro pěstování matematiky*, 98(3):285–287, 1973.
- [7] Wei-Kuo Chiang and Rong-Jaye Chen. Topological properties of the (n, k)-star graph. *International Journal of Foundations of Computer Science*, 9(02):235–248, 1998.
- [8] Yufei Zhao. The number of independent sets in a regular graph. *Combinatorics, Probability and Computing*, 19(2):315–320, 2010.
- [9] Sunil Mathew and MS Sunitha. Cycle connectivity in fuzzy graphs. *Journal of Intelligent & Fuzzy Systems*, 24(3):549–554, 2013.
- [10] Rajab Ali Borzooei and Hossein Rashmanlou. Ring sum in product intuitionistic fuzzy graphs. *Journal of advanced research in pure mathematics*, 7(1):16–31, 2015.
- [11] S Satham Hussain, N Durga, Rahmonlou Hossein, and Ghorai Ganesh. New concepts on quadripartitioned single-valued neutrosophic graph with real-life application. *International Journal of Fuzzy Systems*, 24(3):1515–1529, 2022.
- [12] Takaaki Fujita and Florentin Smarandache. Pythagorean, fermatean, and complex turiyam neutrosophic graphs. *SciNexus*, 2:39–63, 2025.
- [13] Takaaki Fujita. Short note of extended hyperplithogenic sets and general extended plithogenic graphs. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, page 57, 2025.
- [14] Takaaki Fujita. Survey of intersection graphs, fuzzy graphs and neutrosophic graphs. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, page 114, 2024.
- [15] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In *Philosophy*, pages 1–141. American Research Press, 1999.
- [16] N Sarala and T Kavitha. (1, 2)-vertex domination in fuzzy line graphs. *International Journal of Engineering, Science and Mathematics*, 5(4):9–20, 2016.
- [17] Fariba Fayazi, F Mahmudi, and A Gholami. Some properties of fuzzy star graph and fuzzy line graph. *Jordan Journal of Mathematics and Statistics*, 13(1):139–151, 2020.
- [18] Venkata Naga Srinivasa Rao Repalle, Keneni Abera Tola, and Maamo Abebe Ashebo. Extended intuitionistic fuzzy line graphs: Theory and properties. In *Coding Theory Essentials*. IntechOpen, 2023.
- [19] Sumera Naz and Muhammad Aslam Malik. Single-valued neutrosophic line graphs. *TWMS Journal of Applied and Engineering Mathematics*, 8(2):483–494, 2018.
- [20] Keneni Abera Tola, VN Srinivasa Rao Repalle, and Mamo Abebe Ashebo. Theory and application of interval-valued neutrosophic line graphs. *Journal of Mathematics*, 2024(1):5692756, 2024.
- [21] D Ajay and P Chellamani. *Fuzzy magic labelling of Neutrosophic path and star graph*. Infinite Study, 2020.
- [22] Florentin Smarandache, WB Kandasamy, and K Ilanthenral. Neutrosophic graphs: A new dimension to graph theory. 2015.
- [23] Takaaki Fujita and Florentin Smarandache. A review of the hierarchy of plithogenic, neutrosophic, and fuzzy graphs: Survey and applications. In *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond (Second Volume)*. Biblio Publishing, 2024.
- [24] Fazeelat Sultana, Muhammad Gulistan, Mumtaz Ali, Naveed Yaqoob, Muhammad Khan, Tabasam Rashid, and Tauseef Ahmed. A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, 14(10):13139–13159, 2023.
- [25] Reinhard Diestel. *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.
- [26] Florentin Smarandache. Ambiguous set (as) is a particular case of the quadripartitioned neutrosophic set (qns). *nidus idearum*, page 16.
- [27] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77–95. Elsevier, 1975.
- [28] Talal Al-Hawary. Complete fuzzy graphs. *International Journal of Mathematical Combinatorics*, 4:26, 2011.
- [29] Muhammad Akram and Noura Omair Alshehri. Intuitionistic fuzzy cycles and intuitionistic fuzzy trees. *The Scientific World Journal*, 2014(1):305836, 2014.
- [30] Vasantha Kandasamy, K Ilanthenral, and Florentin Smarandache. *Neutrosophic graphs: a new dimension to graph theory*. Infinite Study, 2015.
- [31] TM Nishad, Talal Ali Al-Hawary, and B Mohamed Harif. General fuzzy graphs. *Ratio Mathematica*, 47, 2023.
- [32] Gholamreza Haseli, Shabnam Rahnamay Bonab, Mostafa Hajiaghahi-Keshteli, Saeid Jafarzadeh Ghouschi, and Muhammet Deveci. Fuzzy ze-numbers framework in group decision-making using the bcm and cocoso to address sustainable urban transportation. *Information Sciences*, 653:119809, 2024.
- [33] Qinghai Liu, Zhao Zhang, and Zhihua Yu. Cyclic connectivity of star graph. *Discrete Mathematics, Algorithms and Applications*, 3(04):433–442, 2011.
- [34] Muhammad Akram and Wieslaw A. Dudek. Regular bipolar fuzzy graphs. *Neural Computing and Applications*, 21:197–205, 2012.
- [35] A. Nagoor Gani. On regular fuzzy graphs. 2008.
- [36] Alain Bretto. Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, 1, 2013.
- [37] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558–3565, 2019.
- [38] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.
- [39] Mohammed Alqahtani. Intuitionistic fuzzy quasi-supergraph integration for social network decision making. *International Journal of Analysis and Applications*, 23:137–137, 2025.
- [40] Takaaki Fujita. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*. Biblio Publishing, 2025.

Disclaimer/Publisher's Note

The perspectives, opinions, and data shared in all publications are the sole responsibility of the individual authors and contributors and do not necessarily reflect the views of Sciences Force or the editorial team. Sciences Force and the editorial team disclaim any liability for potential harm to individuals or property resulting from the ideas, methods, instructions, or products referenced in the content.