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The Limits of Symbolic 2-plithogenic

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Abstract

This study is devoted to the analysis of symbolic 2-plithogenic limits and the development of systematic procedures for their computation. In particular, the plithogenic factorization approach and the plithogenic rationalization approach are examined as effective techniques for evaluating such limits. The accuracy of the obtained expressions is further confirmed through the application of L'Hôpital's rule. Additionally, several special stats of symbolic 2-plithogenic limits, including trigonometric forms, are established and analyzed. The theoretical findings are supported and clarified by solving carefully selected numerical examples.

Keywords: Rationalization; Limits; Plithogenic; Factorization; Symbolic.

1 | Introduction

In 2017, Smarandache presented plithogenic logic, which is a sophisticated extension of classical, fuzzy, intuitionistic fuzzy, and neutrosophic logics. It is predicated on the notion that real-world systems frequently exhibit a variety of characteristics, some of which may be incompatible or partially interdependent. Plithogenic logic incorporates a wide range of attribute values, each linked to varying degrees of appurtenance and impacted by dominance and contradiction relationships, in contrast to conventional methods that depend on single-valued truth or membership functions. This paradigm provides a strong tool for modeling complexity, uncertainty, and inconsistency in a variety of domains, including data analysis, artificial intelligence, and decision-making. Plithogenic logic offers a more accurate depiction of human thinking and complicated situations by taking into account the interaction of several qualities and their conflicts. Its uses include risk assessment, medical diagnosis, and multi-criteria decision support, where it is necessary to reconcile contradicting data in order to arrive at well-informed findings. Consequently, plithogenic logic surpasses the capabilities of current fuzzy and neutrosophic frameworks, making it one of the most complete methods for reasoning under uncertainty. We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic set, logic, probability, and statistics [1], in addition to presenting



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introduction to the symbolic plithogenic algebraic structures (revisited), through which he discussed several ideas, including mathematical operations on plithogenic numbers [2]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7]. Also, many other extensions and algorithms in cryptography were presented in [8-9].

2| Main Discussion

2.1| A Factorization Approach to the Limits of Symbolic 2-plithogenic Rational Functions

Consider the rational symbolic 2-plithogenic function:

$$\frac{f(x, P_1, P_2)}{g(x, P_1, P_2)}$$

If $f(x, P_1, P_2)$ and $g(x, P_1, P_2)$ share common factors, these factors may be algebraically canceled (whenever defined), and the limit is evaluated using the simplified form of the function.

Example 1

$$\lim_{x \rightarrow 1+2P_1+2P_2} \frac{x-1-2P_1-2P_2}{x^2-1-8P_1-16P_2}$$

Solution:

$$\lim_{x \rightarrow 1+2P_1+2P_2} \frac{x-1-2P_1-2P_2}{x^2-1-8P_1-16P_2} = \frac{0}{0}$$

Method1:

$$\begin{aligned} \lim_{x \rightarrow 1+2P_1+2P_2} \frac{x-1-2P_1-2P_2}{x^2-1-8P_1-16P_2} &= \lim_{x \rightarrow 1+2P_1+2P_2} \frac{x-1-2P_1-2P_2}{(x-1-2P_1-2P_2)(x+1+2P_1+2P_2)} \\ &= \lim_{x \rightarrow 1+2P_1+2P_2} \frac{1}{x+1+2P_1+2P_2} = \frac{1}{2+4P_1+4P_2} = \frac{1}{2} - \frac{1}{3}P_1 - \frac{1}{15}P_2 \end{aligned}$$

Method2 (L'Hôpital's rule)

$$\begin{aligned} \lim_{x \rightarrow 1+2P_1+2P_2} \frac{x-1-2P_1-2P_2}{x^2-1-8P_1-16P_2} &= \lim_{x \rightarrow 1+2P_1+2P_2} \frac{1}{2x} \\ &= \frac{1}{2+4P_1+4P_2} = \frac{1}{2} - \frac{1}{3}P_1 - \frac{1}{15}P_2 \end{aligned}$$

2.2| A Rationalization Approach to the Limits of Symbolic 2-plithogenic Rational Functions

Example 2:

$$\lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sqrt{1-(8+2P_1+12P_2)x} - \sqrt{1+(8+2P_1+12P_2)x}}{(2+4P_1+4P_2)x}$$

Solution:

$$\lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sqrt{1-(8+2P_1+12P_2)x} - \sqrt{1+(8+2P_1+12P_2)x}}{(2+4P_1+4P_2)x} = \frac{0}{0}$$

Method1:

$$\begin{aligned}
& \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sqrt{1 - (8 + 2P_1 + 12P_2)x} - \sqrt{1 + (8 + 2P_1 + 12P_2)x}}{(2 + 4P_1 + 4P_2)x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sqrt{1 - (8 + 2P_1 + 12P_2)x} - \sqrt{1 + (8 + 2P_1 + 12P_2)x}}{(2 + 4P_1 + 4P_2)x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(\sqrt{1 - (8 + 2P_1 + 12P_2)x} - \sqrt{1 + (8 + 2P_1 + 12P_2)x})(\sqrt{1 - (8 + 2P_1 + 12P_2)x} + \sqrt{1 + (8 + 2P_1 + 12P_2)x})}{(2 + 4P_1 + 4P_2)(\sqrt{1 - (8 + 2P_1 + 12P_2)x} + \sqrt{1 + (8 + 2P_1 + 12P_2)x})x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{1 - (8 + 2P_1 + 12P_2)x - [1 + (8 + 2P_1 + 12P_2)x]}{(2 + 4P_1 + 4P_2)(\sqrt{1 - (8 + 2P_1 + 12P_2)x} + \sqrt{1 + (8 + 2P_1 + 12P_2)x})x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{-(16 + 4P_1 + 24P_2)x}{(2 + 4P_1 + 4P_2)(\sqrt{1 - (8 + 2P_1 + 12P_2)x} + \sqrt{1 + (8 + 2P_1 + 12P_2)x})x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{16 + 4P_1 + 24P_2}{(2 + 4P_1 + 4P_2)(\sqrt{1 - (8 + 2P_1 + 12P_2)x} + \sqrt{1 + (8 + 2P_1 + 12P_2)x})} \\
&= \frac{-8 - 2P_1 - 12P_2}{2 + 4P_1 + 4P_2} = -4 + \frac{7}{3}P_1 - \frac{8}{15}P_2
\end{aligned}$$

Method2 (L'Hôpital's rule)

$$\begin{aligned}
& \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sqrt{1 - (8 + 2P_1 + 12P_2)x} - \sqrt{1 + (8 + 2P_1 + 12P_2)x}}{(2 + 4P_1 + 4P_2)x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\frac{-(8 + 2P_1 + 12P_2)}{2\sqrt{1 - (8 + 2P_1 + 12P_2)x}} - \frac{(8 + 2P_1 + 12P_2)}{2\sqrt{1 + (8 + 2P_1 + 12P_2)x}}}{2 + 4P_1 + 4P_2} \\
&= \frac{\frac{-(8 + 2P_1 + 12P_2)}{2} - \frac{(8 + 2P_1 + 12P_2)}{2}}{2 + 4P_1 + 4P_2} \\
&= \frac{-8 - 2P_1 - 12P_2}{2 + 4P_1 + 4P_2} = -4 + \frac{7}{3}P_1 - \frac{8}{15}P_2
\end{aligned}$$

Example 3

$$\lim_{x \rightarrow 2-3P_1-4P_2} \frac{1 - \sqrt{x - 1 + 3P_1 + 4P_2}}{x - 2 + 3P_1 + 4P_2}$$

Solution:

$$\lim_{x \rightarrow 2-3P_1-4P_2} \frac{1 - \sqrt{x-1+3P_1+4P_2}}{x-2+3P_1+4P_2} = \frac{0}{0}$$

Method1:

$$\begin{aligned} \lim_{x \rightarrow 2-3P_1-4P_2} \frac{1 - \sqrt{x-1+3P_1+4P_2}}{x-2+3P_1+4P_2} \\ &= \lim_{x \rightarrow 2-3P_1-4P_2} \frac{(1 - \sqrt{x-1+3P_1+4P_2})(1 + \sqrt{x-1+3P_1+4P_2})}{(x-2+3P_1+4P_2)(1 + \sqrt{x-1+3P_1+4P_2})} \\ &= \lim_{x \rightarrow 2-3P_1-4P_2} \frac{-x+2-3P_1-4P_2}{(x-2+3P_1+4P_2)(1 + \sqrt{x-1+3P_1+4P_2})} \\ &= \lim_{x \rightarrow 2-3P_1-4P_2} \frac{-(x-2+3P_1+4P_2)}{(x-2+3P_1+4P_2)(1 + \sqrt{x-1+3P_1+4P_2})} \\ &= \lim_{x \rightarrow 2-3P_1-4P_2} \frac{-1}{1 + \sqrt{x-1+3P_1+4P_2}} = \frac{-1}{2} \end{aligned}$$

Method2 (L'Hôpital's rule)

$$\begin{aligned} \lim_{x \rightarrow 2-3P_1-4P_2} \frac{1 - \sqrt{x-1+3P_1+4P_2}}{x-2+3P_1+4P_2} \\ &= \lim_{x \rightarrow 2-3P_1-4P_2} \frac{-1}{2\sqrt{x-1+3P_1+4P_2}} = \frac{-1}{2} \end{aligned}$$

2.3 | Symbolic 2-plithogenic trigonometric limits

$$1) \lim_{x \rightarrow 0+0P_1+0P_2} \sin(l_0 + l_1P_1 + l_2P_2)x = 0$$

$$2) \lim_{x \rightarrow 0+0P_1+0P_2} \cos(l_0 + l_1P_1 + l_2P_2)x = 1$$

$$3) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin(l_0 + l_1P_1 + l_2P_2)x}{x} = l_0 + l_1P_1 + l_2P_2$$

Proof 3

$$\text{put } (l_0 + l_1P_1 + l_2P_2)x = z \quad \Rightarrow \quad x = \frac{1}{l_0 + l_1P_1 + l_2P_2} z$$

$$\text{when } x \rightarrow 0 + 0P_1 + 0P_2 \Rightarrow z \rightarrow 0 + 0P_1 + 0P_2$$

$$\Rightarrow \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin(l_0 + l_1P_1 + l_2P_2)x}{x} = \lim_{z \rightarrow 0+0P_1+0P_2} \frac{\sin z}{\frac{1}{l_0 + l_1P_1 + l_2P_2} z}$$

$$= (l_0 + l_1P_1 + l_2P_2) \lim_{z \rightarrow 0+0P_1+0P_2} \frac{\sin z}{z} = l_0 + l_1P_1 + l_2P_2$$

$$4) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{x}{\sin(l_0 + l_1P_1 + l_2P_2)x} = \frac{1}{l_0 + l_1P_1 + l_2P_2}$$

$$= \frac{1}{l_0} + \left[\frac{1}{l_0 + l_1} - \frac{1}{l_0} \right] P_1 - \left[\frac{1}{l_0 + l_1 + l_2} - \frac{1}{l_0 + l_1} \right] P_2$$

Where l_0, l_1, l_2 are real coefficients, $l_0 \neq 0, l_0 + l_1 \neq 0, l_0 + l_1 + l_2 \neq 0$

Proof 4

$$\text{put } (l_0 + l_1P_1 + l_2P_2)x = z \quad \Rightarrow \quad x = \frac{1}{l_0 + l_1P_1 + l_2P_2} z$$

$$\text{when } x \rightarrow 0 + 0P_1 + 0P_2 \Rightarrow z \rightarrow 0 + 0P_1 + 0P_2$$

$$\Rightarrow \lim_{x \rightarrow 0+0P_1+0P_2} \frac{x}{\sin(l_0 + l_1P_1 + l_2P_2)x} = \lim_{z \rightarrow 0+0P_1+0P_2} \frac{\frac{1}{l_0 + l_1P_1 + l_2P_2} z}{\sin z}$$

$$= \frac{1}{l_0 + l_1P_1 + l_2P_2} \lim_{z \rightarrow 0+0P_1+0P_2} \frac{\sin z}{z}$$

$$= \frac{1}{l_0 + l_1P_1 + l_2P_2}$$

$$= \frac{1}{l_0} + \left[\frac{1}{l_0 + l_1} - \frac{1}{l_0} \right] P_1 - \left[\frac{1}{l_0 + l_1 + l_2} - \frac{1}{l_0 + l_1} \right] P_2$$

$$5) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\tan(l_0 + l_1P_1 + l_2P_2)x}{x} = l_0 + l_1P_1 + l_2P_2$$

$$6) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{x}{\tan(l_0 + l_1P_1 + l_2P_2)x} = \frac{1}{l_0 + l_1P_1 + l_2P_2}$$

$$= \frac{1}{l_0} + \left[\frac{1}{l_0 + l_1} - \frac{1}{l_0} \right] P_1 - \left[\frac{1}{l_0 + l_1 + l_2} - \frac{1}{l_0 + l_1} \right] P_2$$

Where l_0, l_1, l_2 are real coefficients, $l_0 \neq 0, l_0 + l_1 \neq 0, l_0 + l_1 + l_2 \neq 0$, by using the same method in 3, 4, we prove 5 and 6.

Example 5

$$\begin{aligned} 1) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin(4 + P_1 + 7P_2)x}{(5 + 2P_1 + 3P_2)x} &= \frac{4 + P_1 + 7P_2}{5 + 2P_1 + 3P_2} \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin(4 + P_1 + 7P_2)x}{(4 + P_1 + 7P_2)x} \\ &= \frac{4 + P_1 + 7P_2}{5 + 2P_1 + 3P_2} = \frac{4}{5} - \frac{3}{35}P_1 + \frac{17}{35}P_2 \end{aligned}$$

$$\begin{aligned} 3) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\tan(5 + P_1 + P_2)x}{\sin(5 + P_1 + 4P_2)x} &= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\frac{\tan(5 + P_1 + P_2)x}{x}}{\frac{\sin(5 + P_1 + 4P_2)x}{x}} \\ &= \frac{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{\tan(5 + P_1 + P_2)x}{x}}{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin(5 + P_1 + 4P_2)x}{x}} \\ &= \frac{5 + P_1 + P_2}{5 + P_1 + 4P_2} = 1 - \frac{3}{10}P_2 \end{aligned}$$

$$\begin{aligned} 4) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{1 - \cos(-1 + P_1 + P_2)x}{2x^2} &= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\sin^2(-1 + P_1 + P_2)x}{x^2} \\ &= \lim_{x \rightarrow 0+0P_1+0P_2} \left(\frac{\sin(-1 + P_1 + P_2)x}{x} \right)^2 = 1 - P_1 + P_2 \end{aligned}$$

$$\begin{aligned} 5) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(1 + P_1 + P_2)x - \tan(2 + 4P_1 + 3P_2)x}{(3 + P_1 + P_2)x} \\ &= \lim_{x \rightarrow 0+0P_1+0P_2} \left(\frac{(1 + P_1 + P_2)x}{(3 + P_1 + P_2)x} - \frac{\tan(2 + 4P_1 + 3P_2)x}{(3 + P_1 + P_2)x} \right) \\ &= \lim_{x \rightarrow 0+0P_1+0P_2} \left(\frac{1 + P_1 + P_2}{3 + P_1 + P_2} - \left(\frac{2 + 4P_1 + 3P_2}{3 + P_1 + P_2} \right) \frac{\tan(2 + 4P_1 + 3P_2)x}{(2 + 4P_1 + 3P_2)x} \right) \\ &= \frac{1 + P_1 + P_2}{3 + P_1 + P_2} - \left(\frac{2 + 4P_1 + 3P_2}{3 + P_1 + P_2} \right) \\ &= \frac{-1 - 3P_1 - 2P_2}{3 + P_1 + P_2} = \frac{-1}{3} + \frac{4}{3}P_1 - \frac{1}{5}P_2 \end{aligned}$$

2.4 | Some special limits of symbolic 2-plithogenic

$$1) \lim_{x \rightarrow 0+0P_1+0P_2} e^{(l_0+l_1P_1+l_2P_2)x} = 1$$

$$2) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{e^{(l_0+l_1P_1+l_2P_2)x} - 1}{x} = l_0 + l_1P_1 + l_2P_2$$

Proof 2

$$\text{put } (l_0 + l_1 P_1 + l_2 P_2)x = z \quad \Rightarrow \quad x = \frac{1}{l_0 + l_1 P_1 + l_2 P_2} z$$

$$\text{when } x \rightarrow 0 + 0P_1 + 0P_2 \Rightarrow z \rightarrow 0 + 0P_1 + 0P_2$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 0 + 0P_1 + 0P_2} \frac{e^{(l_0 + l_1 P_1 + l_2 P_2)x} - 1}{x} &= \lim_{z \rightarrow 0 + 0P_1 + 0P_2} \frac{e^z - 1}{\frac{1}{l_0 + l_1 P_1 + l_2 P_2} z} \\ &= (l_0 + l_1 P_1 + l_2 P_2) \lim_{z \rightarrow 0 + 0P_1 + 0P_2} \frac{e^z - 1}{z} = (l_0 + l_1 P_1 + l_2 P_2)(1) = l_0 + l_1 P_1 + l_2 P_2 \end{aligned}$$

$$3) \lim_{x \rightarrow 0 + 0P_1 + 0P_2} \frac{\ln(1 + (l_0 + l_1 P_1 + l_2 P_2)x)}{x} = l_0 + l_1 P_1 + l_2 P_2$$

Proof 3

$$\text{Put } (l_0 + l_1 P_1 + l_2 P_2)x = z \quad \Rightarrow \quad x = \frac{1}{l_0 + l_1 P_1 + l_2 P_2} z$$

$$\text{when } x \rightarrow 0 + 0P_1 + 0P_2 \Rightarrow z \rightarrow 0 + 0P_1 + 0P_2$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 0 + 0P_1 + 0P_2} \frac{\ln(1 + (l_0 + l_1 P_1 + l_2 P_2)x)}{x} &= \lim_{z \rightarrow 0 + 0P_1 + 0P_2} \frac{\ln(1 + z)}{\frac{1}{l_0 + l_1 P_1 + l_2 P_2} z} \\ &= (l_0 + l_1 P_1 + l_2 P_2) \lim_{z \rightarrow 0 + 0P_1 + 0P_2} \frac{\ln(1 + z)}{z} = (l_0 + l_1 P_1 + l_2 P_2)(1) \\ &= l_0 + l_1 P_1 + l_2 P_2 \end{aligned}$$

$$4) \lim_{x \rightarrow 0 + 0P_1 + 0P_2} \frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{x} = \ln(l_0 + l_1 P_1 + l_2 P_2)$$

$$= \ln l_0 + [\ln(l_0 + l_1) - \ln l_0]P_1 + [\ln(l_0 + l_1 + l_2) - \ln(l_0 + l_1)]P_2$$

where: $l_0 > 0, l_0 + l_1 > 0, l_0 + l_1 + l_2 > 0$

Proof 4

$$\text{Put } (l_0 + l_1 P_1 + l_2 P_2)^x - 1 = z \quad \Rightarrow \quad (l_0 + l_1 P_1 + l_2 P_2)^x = z + 1$$

$$\ln(l_0 + l_1 P_1 + l_2 P_2)^x = \ln(1 + z)$$

$$x \ln(l_0 + l_1 P_1 + l_2 P_2) = \ln(1 + z)$$

$$x = \frac{1}{\ln(l_0 + l_1 P_1 + l_2 P_2)} \ln(1 + z)$$

$$\text{when } x \rightarrow 0 + 0P_1 + 0P_2 \Rightarrow z \rightarrow 0 + 0P_1 + 0P_2$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{x} &= \lim_{z \rightarrow 0+0P_1+0P_2} \frac{z}{\frac{1}{\ln(l_0 + l_1 P_1 + l_2 P_2)} \ln(1 + z)} \\ &= \ln(l_0 + l_1 P_1 + l_2 P_2) \lim_{z \rightarrow 0+0P_1+0P_2} \frac{z}{\ln(1 + z)} = \ln(l_0 + l_1 P_1 + l_2 P_2) (1) = \ln(l_0 + l_1 P_1 + l_2 P_2) \\ &= \ln l + [\ln(l + m) - \ln l]P_1 + [\ln(l + m + n) - \ln(l + m)]P_2 \end{aligned}$$

Corollary 1

$$\begin{aligned} \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{(c + dP_1 + eP_2)^x - 1} &= \frac{\ln(l_0 + l_1 P_1 + l_2 P_2)}{\ln(c + dP_1 + eP_2)} \\ &= \frac{\ln l_0 + [\ln(l_0 + l_1) - \ln l_0]P_1 + [\ln(l_0 + l_1 + l_2) - \ln(l_0 + l_1)]P_2}{\ln c + [\ln(c + d) - \ln c]P_1 + [\ln(c + d + e) - \ln(c + d)]P_2} \end{aligned}$$

where: $l_0 > 0, l_0 + l_1 > 0, l_0 + l_1 + l_2 > 0$ and $c > 0, c + d > 0, c + d + e > 0$

Proof:

$$\begin{aligned} \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{(c + dP_1 + eP_2)^x - 1} &= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{x}}{\frac{(c + dP_1 + eP_2)^x - 1}{x}} \\ &= \frac{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{(l_0 + l_1 P_1 + l_2 P_2)^x - 1}{x}}{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{(c + dP_1 + eP_2)^x - 1}{x}} \\ &= \frac{\ln(l_0 + l_1 P_1 + l_2 P_2)}{\ln(c + dP_1 + eP_2)} = \frac{\ln l_0 + [\ln(l_0 + l_1) - \ln l_0]P_1 + [\ln(l_0 + l_1 + l_2) - \ln(l_0 + l_1)]P_2}{\ln c + [\ln(c + d) - \ln c]P_1 + [\ln(c + d + e) - \ln(c + d)]P_2} \end{aligned}$$

Example 6

$$1) \lim_{t \rightarrow 0+0P_1+0P_2} e^{(2+4P_1+7P_2)t} = 1$$

$$2) \lim_{t \rightarrow 0+0P_1+0P_2} \frac{e^{(1-55P_1+9P_2)t} - 1}{t} = 1 - 55P_1 + 9P_2$$

$$3) \lim_{t \rightarrow 0+0P_1+0P_2} \frac{(1 + 5P_1 + 6P_2)^t - 1}{t} = \ln(1 + 5P_1 + 6P_2)$$

$$= \ln 6 P_1 + [\ln 12 - \ln 6] P_2$$

$$4) \lim_{t \rightarrow 0+0P_1+0P_2} \frac{(3+7P_1+10P_2)^t - 1}{e^{(15-5P_1+11P_2)t} - 1} = \lim_{t \rightarrow 0+0P_1+0P_2} \frac{\frac{(3+7P_1+10P_2)^t - 1}{t}}{\frac{e^{(15-5P_1+11P_2)t} - 1}{t}}$$

$$= \frac{\lim_{t \rightarrow 0+0P_1+0P_2} \frac{(3+7P_1+10P_2)^t - 1}{t}}{\lim_{t \rightarrow 0+0P_1+0P_2} \frac{e^{(15-5P_1+11P_2)t} - 1}{t}} = \frac{\ln(3+7P_1+10P_2)}{15-5P_1+11P_2}$$

$$= \left(\frac{1}{15} + \frac{1}{30} P_1 - \frac{11}{210} P_2 \right) \left(\ln 3 + \ln \frac{10}{3} P_1 + \ln 2 P_2 \right)$$

$$5) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(6+2P_1+P_2)^x - 1}{(6+2P_1+2P_2)^x - 1} = \frac{\ln(6+2P_1+P_2)}{\ln(6+2P_1+2P_2)} = \frac{\ln 6 + [\ln 8 - \ln 6] P_1 + [\ln 9 - \ln 8] P_2}{\ln 6 + [\ln 8 - 6] P_1 + [\ln 10 - \ln 8] P_2}$$

$$= \frac{\ln 2 + \left[\ln \frac{4}{3} \right] P_1 + \left[\ln \frac{9}{8} \right] P_2}{\ln 2 + \left[\ln \frac{4}{3} \right] P_1 + \left[\ln \frac{5}{4} \right] P_2}$$

$$= 1 + \left[\frac{\ln 2 + \ln \frac{4}{3} + \ln \frac{9}{8}}{\ln 2 + \ln \frac{4}{3} + \ln \frac{5}{4}} - \frac{\ln 2 + \ln \frac{4}{3}}{\ln 2 + \ln \frac{4}{3}} \right] P_2$$

$$6) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{1}{x} \ln(1 + (10 + 21P_1 + 77P_2)x) = 10 + 21P_1 + 77P_2$$

$$7) \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\tan(2+4P_1+3P_2)x}{e^{(4+2P_1+P_2)x} - 1} = \lim_{x \rightarrow 0+0P_1+0P_2} \frac{\frac{\tan(2+4P_1+3P_2)x}{x}}{\frac{e^{(4+2P_1+P_2)x} - 1}{x}}$$

$$= \frac{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{\tan(2+4P_1+3P_2)x}{x}}{\lim_{x \rightarrow 0+0P_1+0P_2} \frac{e^{(4+2P_1+P_2)x} - 1}{x}}$$

$$= \frac{2+4P_1+3P_2}{4+2P_1+P_2} = \frac{1}{2} + \frac{1}{2} P_1 + \frac{2}{7} P_2$$

$$\begin{aligned}
8) \quad & \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(1+2P_1+3P_2)^x - (2+4P_1+3P_2)^x}{x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(1+2P_1+3P_2)^x - 1 - (2+4P_1+3P_2)^x + 1}{x} \\
&= \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(1+2P_1+3P_2)^x - 1}{x} - \lim_{x \rightarrow 0+0P_1+0P_2} \frac{(2+4P_1+3P_2)^x - 1}{x} \\
&= \ln(1+2P_1+3P_2) - \ln(2+4P_1+3P_2) = \ln\left(\frac{1+2P_1+3P_2}{2+4P_1+3P_2}\right) \\
&= \ln\left(\frac{1}{2} + \frac{1}{6}P_2\right) = -\ln 2 + \left[\ln \frac{2}{3}\right]P_2
\end{aligned}$$

3 | Conclusion

Limits represent one of the fundamental pillars of calculus, as they provide the essential framework for understanding differentiation through the analysis of infinitesimal behavior. Motivated by this foundational role, the present study introduces and develops the concept of symbolic 2-plithogenic limits. Various analytical techniques for evaluating these limits are systematically examined, together with the formulation of specific rules designed to simplify their computation. Furthermore, the consistency of the proposed approaches is demonstrated by solving illustrative examples using alternative procedures, including the application of L'Hôpital's rule, which confirms the equivalence of the obtained results.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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