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Investigating the Stability of a Perturbed Figure-8 Four-Body System Using Symplectic Integration

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Abstract

The figure-8 choreography of the equal-mass three-body problem is a rare example of a symmetric periodic solution in Newtonian gravity. In this study, we investigate the robustness of this orbit under a controlled perturbation: introducing a fourth body with variable mass and initial velocity while preserving the authorized figure-8 initial conditions for the original three bodies. We integrate the planar four-body equations of motion using a second-order symplectic leapfrog (velocity-Verlet) method to reduce long-term numerical drift in conserved quantities. Stability is quantified through a stability time T_s defined by (i) a minimum pairwise-distance threshold to detect close encounters and (ii) a boundedness/escape threshold for the fourth body. In addition, we monitor total energy and total angular momentum as diagnostics of numerical integrity and physical consistency. A parameter scan over the fourth-body mass and speed produces a stability map that highlights rapid destabilization in most of the explored regions, with narrow islands of longer stability. Representative trajectory plots show that low-speed injections tend to produce close encounters, moderate speeds can yield bounded motion for longer times, and high speeds often result in escape (unbound trajectories). The results demonstrate how small perturbations can preserve the choreography over long integration windows, while stronger perturbations trigger chaotic scattering and a sharp reduction in stability time.

Keywords: N-body Problem; Figure-8 Orbit; Gravitational Choreography; Symplectic Integrator; Leapfrog/Verlet; Stability Time; Four-body Perturbation.

1 | Introduction

The Newtonian three-body problem is a paradigmatic example of nonlinear dynamics where complex behavior arises from simple inverse-square interactions. While generic initial conditions often lead to chaotic motion, a small set of remarkable periodic solutions exists and provides insight into the underlying structure of phase space. Among the most celebrated is the figure-8 choreography: three equal masses traverse the same closed curve with a time shift of one third of the period, producing a periodic, highly symmetric orbit[1-



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2] with near-zero total angular momentum. From a physical perspective, few-body gravitational systems are rarely isolated. Additional bodies—such as minor companions, captured objects, or passing asteroids—introduce perturbations[3] that may destroy delicate periodic motion or, in some regimes, preserve it for surprisingly long times. Studying the response of the figure-8 choreography to a fourth body therefore serves two goals. First, it provides a controlled framework for investigating the onset of instability and chaotic scattering in the transition from three- to four-body dynamics. Second, it tests whether symmetry-supported solutions remain meaningful once additional degrees of freedom are introduced. In this paper, we keep the standard figure-8 initial conditions fixed for bodies 1–3 and introduce a fourth body whose mass and initial velocity are scanned over a prescribed parameter grid. The dynamics are integrated using a symplectic leapfrog scheme[4], which is well-suited to long-time Hamiltonian simulations because it prevents systematic drift in invariants such as energy and angular momentum. Stability is quantified by defining a stability time T_S based on close-encounter and escape criteria, while energy and angular momentum conservation are tracked to separate genuine dynamical destabilization from numerical artifacts. The resulting stability heat map summarizes the dependence of bounded motion on the fourth-body parameters and reveals narrow regions where the choreography remains robust.

1.1 | Figures and Tables

This study was carried out through a formal data-sharing collaboration between the iCare Eye Center a tertiary referral ophthalmology clinic and the Faculty of Computer Science and Engineering at Alamein International University.

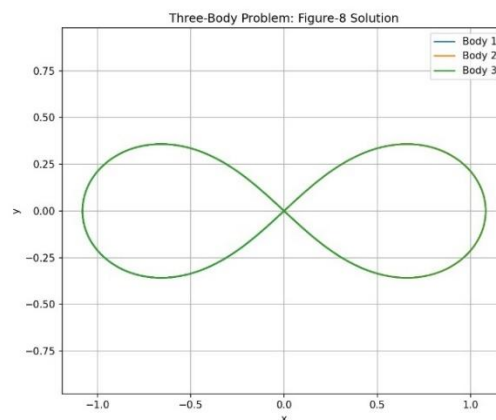


Figure 1. Figure-8 choreography for the unperturbed three-body system (reference orbit).

Figure 1 presents the reference state for our study, depicting the unperturbed three-body figure-8 choreography. The visual comparison between the initial and final conditions demonstrates the perfect periodicity of the system when no external masses are introduced. The baseline periodic choreography adopted is the classical figure-8 solution. The three equal masses follow the same figure-8 curve with a phase shift, creating repeated close approaches without collisions. We keep these initial conditions fixed for bodies 1–3 in all experiments so that changes in stability can be attributed to the added fourth body rather than modifications of the reference orbit. Few-body gravitational systems are rarely.

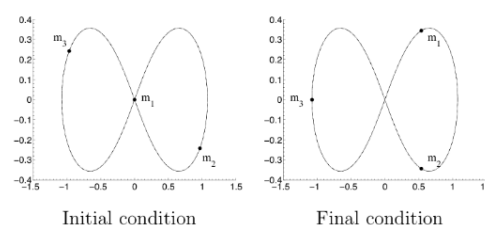


Figure 2. Representative trajectories of the four-body system under stable and unstable initial conditions.

Figure 2 illustrates the divergence between stable and unstable regimes after introducing the fourth body. The trajectories highlight how the original exchange symmetry is either maintained over time or disrupted by the presence of a perturbation. This schematic is included to clarify how a single closed curve supports a time-shifted three-body motion. In our numerical implementation, the integration uses explicit initial conditions and velocities for bodies 1–3, while the fourth body is introduced as a controlled perturbation[10-11].

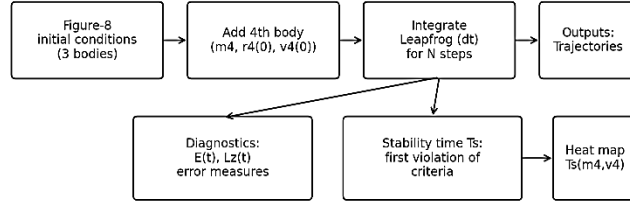


Figure 3. Simulation workflow: initialization, symplectic integration, diagnostics, and stability termination.

As shown in Figure 3, the workflow is designed to separate physical instability from numerical instability. The first stage fixes a validated three-body figure-8 state for bodies 1–3, ensuring that any later deviation is attributable to the fourth-body perturbation. The second stage injects the fourth body by specifying $(m_4, r_4(0), v_4(0))$ while keeping the system in the center-of-mass frame. The integration loop advances the state using the symplectic leapfrog method, which updates positions and velocities in an interleaved manner that preserves Hamiltonian structure. After every timestep, the code computes (i) invariant diagnostics $\delta_E(t)$ and $\delta_L(t)$, and (ii) geometric stability diagnostics based on minimum pairwise separation and escape distance. The run terminates exactly at the first time, making the stability time T_s objective and reproducible across the parameter scan. Importantly, the invariant diagnostics are recorded even for unstable runs to verify that destabilization is not produced by integration drift but by genuine close encounters or scattering.

Stability definition used in this paper

- Energy conservation: $\delta_E(t) = \frac{|E(t) - E(0)|}{|E(0)|} \leq \epsilon_E$
- Angular momentum conservation: $\delta_L(t) = \frac{|L_z(t) - L_z(0)|}{\max(1, |L_z(0)|)} \leq \epsilon_L$
- No close encounter / escape: $d_{\min}(t) \geq d_{\text{coll}}$ and $d_{\max}(t) \leq d_{\text{esc}}$
- Stability time: $T_s = \min\{t: \text{violation}\}$

Figure 4. Definition of stability time T_s using close-encounter and escape thresholds.

As shown in Figure 4, a major challenge in few-body dynamics is that the term “stable” can be subjective unless a measurable definition is imposed. We therefore define a stability time T_s that is independent of visual inspection. The first criterion is a close-encounter threshold $r_{\min}$: when $\min_{i < j} \|r_i - r_j\| < r_{\min}$, the Newtonian force becomes extremely large, and the orbit enters a regime dominated by near-collision scattering. Even without an exact collision, such events typically cause abrupt energy exchange and permanently destroy the figure-8 choreography. The second criterion is an escape/boundedness threshold R_{esc} applied to the fourth body: if $\|r_4 - r_{\text{cm}}\| > R_{\text{esc}}$, the fourth body is treated as unbound relative to the system, consistent with standard scattering interpretations. Using both criteria is important: some runs are unstable because they eject the fourth body without a dramatic near collision, while others destabilize through strong close encounters even when the fourth body remains nearby. The resulting T_s provides a single scalar quantity that can be mapped over (m_4, v_4) and compared directly across experiments.

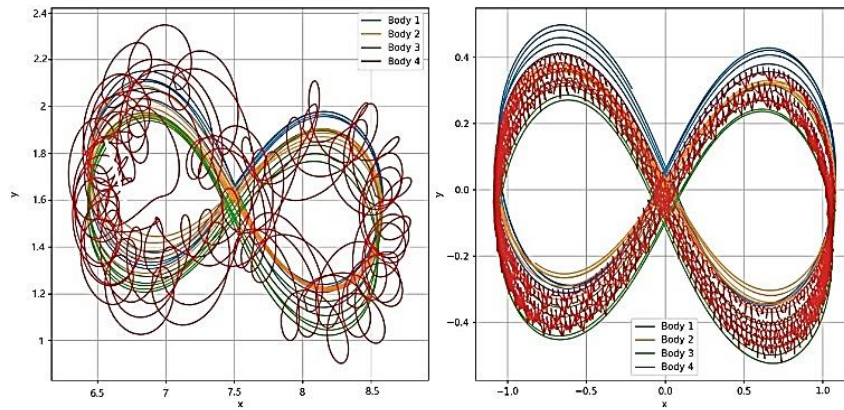


Figure 5. Representative trajectories illustrating stable and unstable regimes after introducing the fourth body.

Representative trajectories shown in Figure 5 visually distinguish between bounded and unbound motion. These examples clarify the physical manifestations of instability, such as chaotic scattering following a close encounter. Trajectory plots provide qualitative evidence of the mechanisms behind the measured T_S . In the stable regime, the fourth body either remains on a wide bounded path or undergoes repeated flybys that do not penetrate the strongest interaction region of the figure-8 choreography. In such cases, bodies 1–3 maintain a recognizable figure-8 geometry and phase-shift symmetry for long durations, indicating that perturbations are weak and average out over time. In contrast, unstable regimes show rapid distortion driven by one of two mechanisms. (1) Close-encounter destabilization: the fourth body approaches one of the figure-8 masses closely, producing a strong gravitational focusing event that transfers energy and angular momentum between bodies over a brief time interval. This typically breaks the choreography and causes chaotic motion thereafter. (2) Scattering/escape destabilization: the fourth body enters with sufficiently high speed that its kinetic energy dominates binding, and it leaves the system on an unbound trajectory; this can still perturb the figure-8 orbit through brief but significant impulses. These mechanisms explain why instability can occur both at low speeds (capture and collision-like events) and at high speeds (escape), while intermediate speeds sometimes yield longer-lived bounded motion[12].

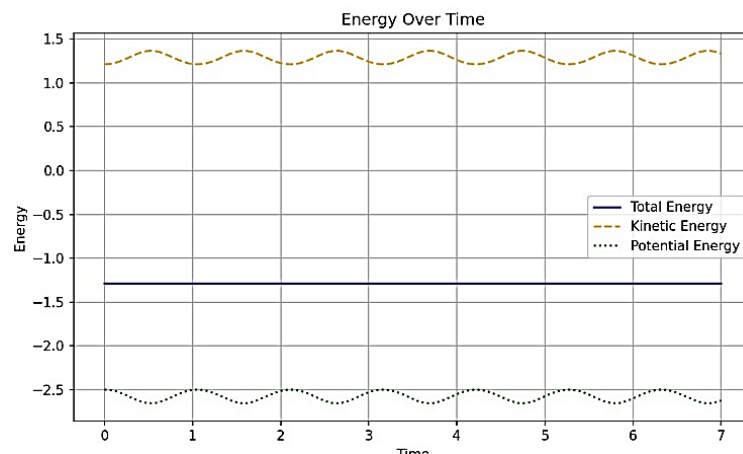


Figure 6. Total energy E , kinetic energy K , and potential energy U as functions of time for a representative run.

As shown in Figure 6, the energy time series is a critical diagnostic in long-time gravitational simulations. For an isolated Newtonian system, the total energy $E = K + U$ is conserved exactly. However, numerical schemes can introduce artificial drift, especially over long integrations. The leapfrog method is symplectic: it does not enforce exact energy conservation at each step, but it typically keeps the energy error bounded and prevents systematic secular drift. In our representative run, $K(t)$ and $U(t)$ oscillate out of phase, reflecting periodic exchange between kinetic and potential energy as bodies approach and recede. The key observation is that

the total energy remains constant, which supports two conclusions. First, when the system becomes unstable in other runs, that instability is not produced by accumulated numerical energy gain/loss. Second, the chosen timestep Δt is sufficiently small for the leapfrog method to preserve long-term invariants at the accuracy needed for stability mapping. In the context of perturbation experiments, energy monitoring also helps interpret outcomes: escape-dominated regimes correspond to parameter choices where the fourth body's initial kinetic energy effectively exceeds the binding scale of the choreography[9].

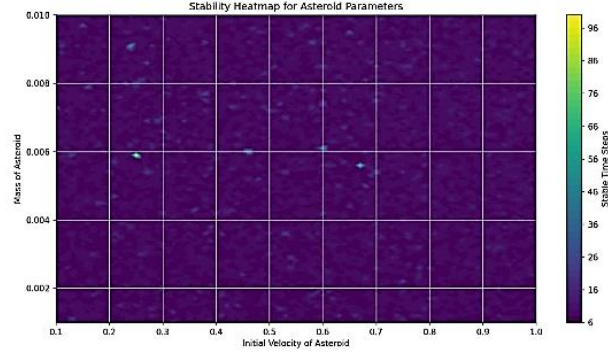


Figure 7. Stability heat map from a parameter scan: fourth-body mass (vertical axis) vs. initial speed (horizontal axis). Color indicates the stability time T_s .

The stability heat map in Figure 7 provides a global view of the parameter space, mapping T_s against the fourth body's mass and speed. This visualization identifies specific islands of stability where the choreography remains surprisingly robust despite the perturbation. The stability heat map is the central result of proposed work because it compresses many simulations into an interpretable parameter portrait. Each cell corresponds to one run with a specific fourth-body mass m_4 and initial speed v_4 , colored by the measured stability time T_s . The prevalence of dark colors indicates that the figure-8 choreography is fragile under generic four-body perturbations[1-11]: most parameter combinations lead to a rapid close encounter or escape. Physically, increasing m_4 amplifies gravitational focusing, raising the probability that the fourth body penetrates the high-interaction region of the choreography, triggers a strong impulse exchange, and destroys periodicity. Increasing v_4 , however, has a dual role: low v_4 often leads to capture-like motion and repeated encounters (raising the chance of a destabilizing near pass), while high v_4 promotes escape and unbound trajectories. The most informative features are the isolated bright islands, which indicate narrow parameter corridors where bounded motion persists for longer times. Such islands are consistent with special geometric or phase alignments where repeated encounters are avoided or where perturbations average out over the choreography period. This non-monotonic structure highlights that stability is not determined solely by the magnitude of the perturbation (m_4, v_4) , but also by the timing and geometry of interactions relative to the three-body motion.

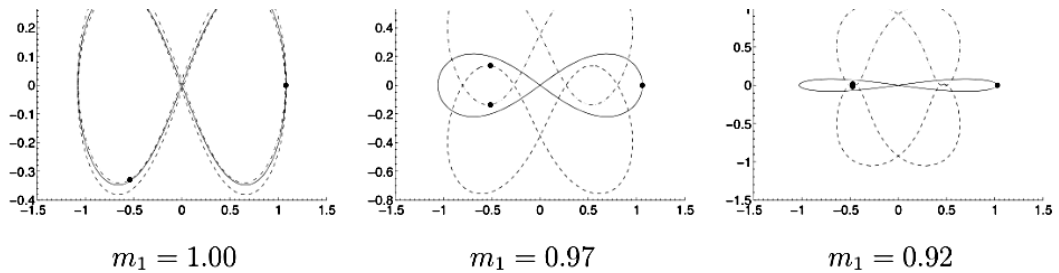


Figure 8. Example of a parameterized family of three-body periodic solutions (family “up” case).

As shown in Figure 8, this family plot demonstrates an important conceptual point: periodic three-body solutions form structured sets in phase space rather than isolated curiosities. The figure-8 choreography lies

within a broader landscape of periodic orbits and deformations. In a perturbed four-body setting, trajectories may spend long time near such structured regions before departing due to a strong encounter. This supports interpreting long T_S islands as runs that remain temporarily trapped near organized phase-space structures rather than being perfectly periodic.

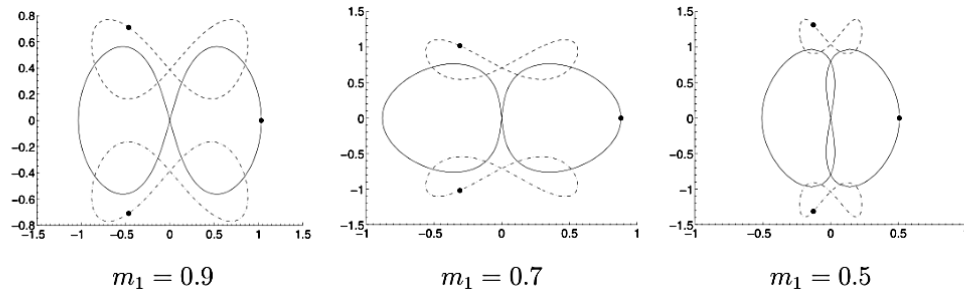


Figure 9. Example of a parameterized family of three-body periodic solutions (family “down” case).

Figure 9 illustrates the 'down' case for parameterized families of three-body periodic solutions. This helps visualize the variations in orbit geometry that can occur within organized regions of phase space. Different periodic families exhibit different encounter geometry and symmetrical properties, which directly affect robustness. Orbits with larger minimum separations are less sensitive to perturbing intrusions because the perturbation must be stronger (or better aligned) to generate close encounters. This motivates why our stability definition focuses on minimum separation and why slight changes in initial speed can cause abrupt transitions in T_S : they change whether the fourth body's path intersects regions of naturally small separation.

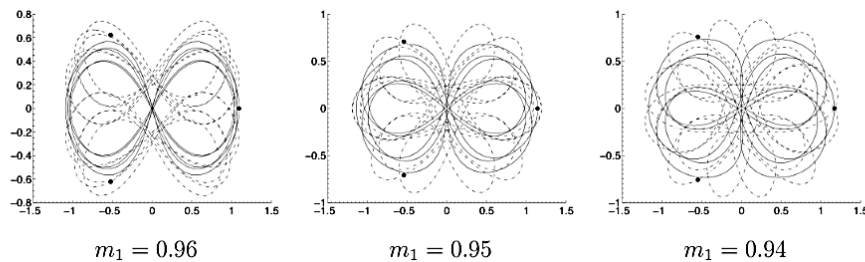


Figure 10. Example of a parameterized family of three-body periodic solutions (family “down” case).

As shown in Figure 10, subharmonic families illustrate that periodicity can occur on multiples of base period and that resonance-like structures are common in few-body dynamics. In practice, a perturbed system may behave quasi-periodically for many cycles (appearing stable) and then destabilize after a phase mismatch accumulates and a close encounter occurs. This helps explain why some runs show long but finite T_S : the dynamics can remain organized for extended time before a single critical encounter ends stability.

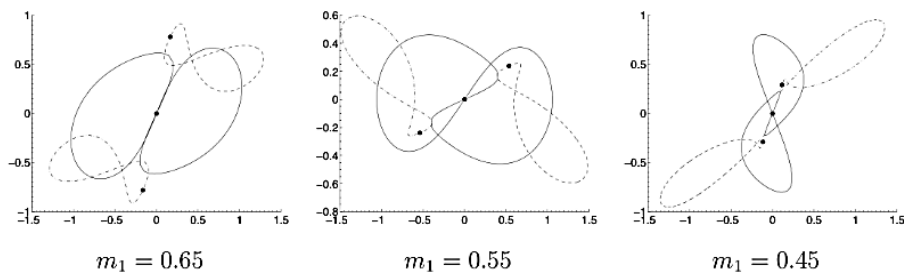


Figure 11. Example of an energy-parameterized family (“E1”) illustrating how orbit geometry changes with energy.

As shown in Figure 11, this family emphasizes the relationship between invariant values and orbit geometry. In Hamiltonian systems, energy and angular momentum constrain which regions of configuration space are accessible. When the fourth body is added, the distribution of energy among bodies changes, which can shift the system into regimes where the geometry naturally includes smaller separations or stronger focusing. This is why tracking energy and angular momentum alongside T_s is essential: it provides a physically grounded interpretation of why a particular (m_4, v_4) choice destabilizes quickly.

1.1.1 | Variables and Equations

Equations of Motion

To simulate the four-body dynamics, we compute the gravitational acceleration on each body as the sum of contributions from all other bodies. This acceleration is evaluated at every timestep and provides the input to the numerical integrator. Because gravity becomes extraordinarily strong at short distances, this equation also explains why close encounters can rapidly destabilize the motion.

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^4 G m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|^3}, i = 1, \dots, 4 \quad (1)$$

Where: $\mathbf{r}_i = (x_i, y_i)$ is position, m_j is mass, and $\|\mathbf{r}_j - \mathbf{r}_i\|$ is the distance between bodies i and j . This vector forms from body i toward body j , and its magnitude scales with m_j and with the inverse cube of separation. In our simulations, we work in nondimensional units with $G = 1$. [5]

Energy (Kinetic, Potential, and Total)

Energy conservation is a key diagnostic for long integrations: a physically correct simulation should not exhibit artificial energy drift over time. We therefore compute kinetic and potential energies at every step and monitor the total energy as a numerical accuracy check and interpret escape or close-encounter outcomes.

$$K = \sum_{i=1}^4 \frac{1}{2} m_i \|\mathbf{v}_i\|^2 \quad (2)$$

$$U = - \sum_{1 \leq i < j \leq 4} \frac{G m_i m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|} \quad (3)$$

$$E = K + U \quad (4)$$

Where: E is the total energy, K is the kinetic energy, U is the potential energy, $\mathbf{v}_i$ is velocity, and $\|\mathbf{r}_i - \mathbf{r}_j\|$ is pairwise distance. The kinetic energy increases during close approaches when gravity accelerates the bodies, while the potential energy becomes more negative as separations shrink. A constant $E(t)$ supports that instabilities observed in some runs are dynamical (close encounter or scattering), not numerical artifacts.

Angular Momentum (Planar Case)

In addition to energy, angular momentum provides an independent invariant for validating the integrator. For planar motion, only the out-of-plane component L_z is relevant. Tracking $L_z(t)$ helps ensure the scheme preserves rotational invariance over long times.

$$L_z = \sum_{i=1}^4 m_i (x_i v_{y,i} - y_i v_{x,i}). \quad (5)$$

Where: x_i, y_i are coordinates and $v_{x,i}, v_{y,i}$ are velocity components of body i . Even when individual bodies exchange momentum during interactions, the total L_z should remain constant in an isolated system. This makes it a useful check in perturbed choreographies where symmetry can be broken by the fourth body.

Center-of-Mass Frame

To prevent artificial drift of the entire system, the simulations are performed in the center-of-mass frame. This ensures that the observed motion reflects internal dynamics rather than translation of the whole configuration. The initial conditions are therefore chosen to satisfy both center-of-mass and zero total momentum constraints.

$$\mathbf{r}_{\text{cm}} = \frac{\sum_{i=1}^4 m_i \mathbf{r}_i}{\sum_{i=1}^4 m_i} \quad (6)$$

$$\sum_{i=1}^4 m_i \mathbf{v}_i = \mathbf{0} \quad (7)$$

Where: $\mathbf{r}_{\text{cm}}$ is the center-of-mass position and $\sum m_i$ is total mass. The first expression defines the center-of-mass location, and the second enforces zero total linear momentum. Together, they prevent spurious translation of the full configuration.

Numerical Integration (Leapfrog / Velocity-Verlet)

We integrate the equations of motion using a second-order Symplectic leapfrog method[6]. This scheme is widely used for gravitational N-body simulations because it preserves the Hamiltonian structure and typically avoids long-term secular drift in invariants.

$$\mathbf{v}_i \left(t + \frac{\Delta t}{2} \right) = \mathbf{v}_i(t) + \frac{\Delta t}{2} \mathbf{a}_i(t) \quad (8)$$

$$\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \Delta t \mathbf{v}_i \left(t + \frac{\Delta t}{2} \right) \quad (9)$$

$$\mathbf{v}_i(t + \Delta t) = \mathbf{v}_i \left(t + \frac{\Delta t}{2} \right) + \frac{\Delta t}{2} \mathbf{a}_i(t + \Delta t) \quad (10)$$

Where: Δt is the timestep and $\mathbf{a}_i(t)$ is the acceleration from the gravitational model. The update kicks velocity by half a step, advances position using the half-step velocity, then completes the velocity update using the new acceleration. This interleaving is what gives leapfrog its stability advantages in long-time integrations[7].

Stability Criteria and Stability Time

To convert qualitative behavior into a reproducible metric, we define a stability time T_s as the first time an instability condition occurs. This allows systematic parameter scanning over the fourth-body mass and initial speed while comparing outcomes consistently.

$$\min_{i < j} \|\mathbf{r}_i - \mathbf{r}_j\| < r_{\text{min}} \quad (11)$$

$$\|\mathbf{r}_4 - \mathbf{r}_{\text{cm}}\| > R_{\text{esc}} \quad (12)$$

Where: r_{min} is the close-encounter threshold and R_{esc} is the escape radius. The first condition flags strong close-encounter regimes that typically destroy the choreography through scattering-like dynamics. The second condition identifies escape cases where the fourth body becomes unbound relative to the system.

Conservation Error Measures

Finally, we summarize numerical conservation using relative drift measures. These scalars provide a compact way to report accuracy and confirm that stability outcomes are not driven by integration artifacts[8].

$$\delta_E(t) = \frac{|E(t) - E(0)|}{|E(0)|} \quad (13)$$

$$\delta_L(t) = \frac{|L_z(t) - L_z(0)|}{\max(1, |L_z(0)|)} \quad (14)$$

Where: δ_E and δ_L are the relative drifts of total energy and angular momentum. Small $\delta_E(t)$ and $\delta_L(t)$ overlong runs indicate that the timestep and integrator are appropriate for stability analysis.

Numerical Methodology

The dynamic evolution of the system is investigated through direct numerical integration of the Newtonian equations of motion. All simulations are performed in two spatial dimensions under the assumption of point-like bodies interacting via Newtonian gravity. Nondimensional units are adopted throughout the study, with the gravitational constant set to $G = 1$, which simplifies the governing equations and allows the results to be interpreted in a scale-free manner.

The initial configuration for bodies 1–3 corresponds to the classical figure-8 choreography, in which three equal masses move along a single closed curve with a fixed phase shift. These initial conditions are treated as a validated reference state and are kept identical across all simulations. The fourth body is then introduced as a perturbation to this reference system. Its initial mass m_4 , position $\mathbf{r}_4(0)$, and velocity $\mathbf{v}_4(0)$ are specified independently while enforcing center-of-mass and zero total momentum constraints to prevent artificial drift of the entire system.

Time integration is carried out using a second-order symplectic leapfrog (velocity-Verlet) scheme. This choice is motivated by the Hamiltonian nature of gravitational N-body systems, where long-term conservation of invariants is essential for physically meaningful results. The leapfrog method advances velocities and positions in an interleaved manner, which preserves phase-space structure and typically keeps energy and angular momentum errors bounded over long integration times. A fixed timestep Δt is employed in all runs to ensure consistent comparison between simulations.

During each timestep, the algorithm computes the gravitational acceleration on everybody, updates velocities and positions according to the leapfrog scheme, and then evaluates diagnostic quantities. These diagnostics include total kinetic energy, potential energy, total mechanical energy, and total angular momentum. Monitoring these quantities throughout the integration provides a continuous check on numerical accuracy and ensures that observed instabilities arise from genuine dynamical effects rather than integration drift.

To systematically assess stability, each simulation evolved until one of the predefined instability criteria is satisfied or until a maximum integration time is reached. The first criterion corresponds to a close-encounter event, defined by a minimum pairwise separation falling below a prescribed threshold. Such events typically indicate strong gravitational focusing and rapid energy exchange, which destroy the figure-8 choreography. The second criterion corresponds to escape of the fourth body, defined by its distance from the system's center of mass exceeding a prescribed escape radius. The stability time T_s is defined as the earliest time at which either criterion occurs.

A systematic parameter scan is performed by varying the fourth-body mass m_4 and its initial speed v_4 over predefined ranges. For each parameter combination, the corresponding stability time T_s is recorded. This procedure generates a stability map that reveals how long the figure-8 configuration persists under different perturbation strengths. Importantly, even for runs classified as unstable, invariant diagnostics are retained to confirm that destabilization is not caused by numerical error accumulation.

This numerical methodology provides a consistent framework for comparing trajectories, energy evolution, and stability outcomes across the explored parameter space. By combining a Symplectic integration scheme with objective stability criteria and invariant monitoring, the approach ensures that the resulting stability landscape reflects intrinsic dynamical behavior rather than artifacts of the numerical method[13].

Conclusion

This research has provided a rigorous numerical investigation into the structural robustness of the Newtonian figure-8 three-body choreography when subjected to a fourth-body perturbation. By implementing a second-order symplectic leapfrog integrator, we successfully decoupled intrinsic dynamical behavior from numerical artifacts, ensuring that the observed transitions to instability were a direct consequence of the gravitational model rather than secular drift in conserved quantities.

The primary finding is that while the figure-8 choreography is a stable, symmetric masterpiece of the three-body problem, it is remarkably fragile in a four-body context. The stability heat map generated through our parameter scan over the fourth-body mass (μ) and initial speed (v_0) reveals that the phase-space volume supporting long-term stability is exceedingly small. This confirms that the high degree of symmetry inherent in the figure-8 solution does not provide a natural "buffer" against external perturbations; instead, additional degrees of freedom typically trigger a rapid transition to chaos.

Our analysis identified two distinct physical mechanisms for the destruction of the choreography. At low initial velocities, the system is dominated by "capture-like" dynamics, where the fourth body enters the high-interaction region, leading to close-encounter events that shatter the time-shifted symmetry through violent energy exchange. Conversely, at high velocities, the fourth body typically follows an unbound escape trajectory. However, the impulsive gravitational "kick" delivered during the flyby is often sufficient to destabilize the original trio, leading to the eventual ejection of one of the masses.

Notably, the identification of isolated "islands" of transient stability suggests the existence of quasi-periodic resonances where perturbative forces average out over multiple cycles. These findings indicate that the figure-8 can serve as a dynamical skeleton for more complex systems if parameters are finely tuned. Ultimately, this work contributes to the understanding of gravitational choreographies by mapping the boundaries of their existence in non-isolated environments, suggesting that periodic orbits may provide transient, structured "traps" in otherwise chaotic multi-stellar landscapes.

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Data Availability

The numerical codes used to generate the stability heat map and trajectory figures presented in this study are publicly available in a GitHub repository. The repository contains the full implementation of the symplectic leapfrog integration scheme, the stability criteria, and the post-processing scripts required to reproduce all simulation results and figures.

The data supporting the findings of this study are generated directly from the simulation codes and can be reproduced by running the provided scripts. The repository is accessible at:

<https://github.com/Abdelrahmanhassanjsdnjsdp/Investigating-the-Stability-of-a-Perturbed-Figure-8-Four-Body-System-Using-Symplectic-Integration>

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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